Sigma/Gamma/Beta-nought Pros and Cons: Calibration and Use of Terrain-Corrected Products in the Age of Geocoded Single Look Complex Radar Images

Howard Zebker, Stanford University
Francois Charbonneau, Natural Resources Canada
David Small, University of Zurich

Many radar standards/products in common use

- Need to define units
- Need to be able to calibrate

- Radar backscatter usually described:
  - Radar cross section (σ)
  - Radar cross section per range-Doppler area (β^0)
  - Specific (or normalized) radar cross section (σ^0)
  - “Corrected” normalized radar cross section (γ^0)

Radar equation

- Derives received signal power from instrument and viewing geometry

$$P_{signal} = \frac{P_t G_t}{4\pi R^2} \cdot \sigma \cdot \frac{A_r}{4\pi R^2}$$

Received signal power

Radar cross section

- Radar cross section σ is least familiar of these terms as others are easily measurable quantities

Rearrange radar equation

- Isolate radar cross section for definition – rearrange terms first

$$\frac{P_{signal}}{A_r} \frac{4\pi R^2}{P_t G_t} 4\pi R^2 = \sigma$$

- Now restate as ratio of power densities for formal relation as per IEEE

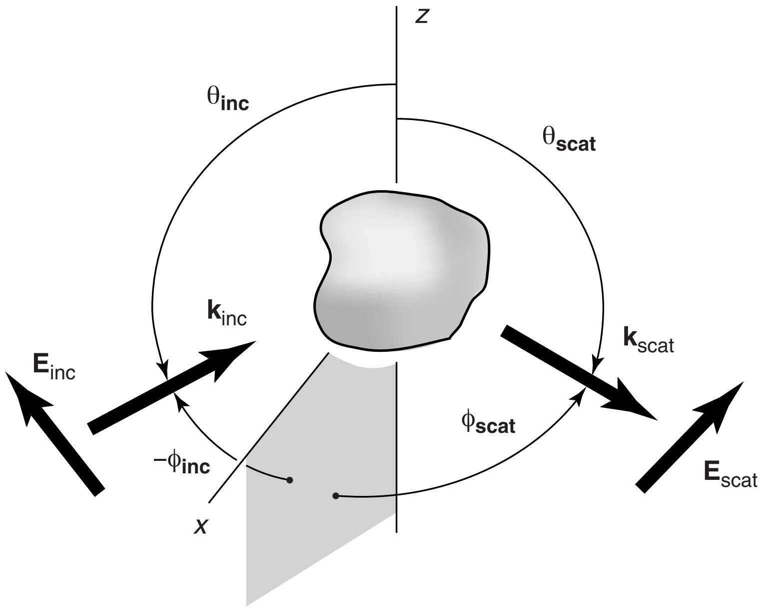
$$|E_{scatt}|^2 \frac{1}{|E_{inc}|^2} 4\pi R^2 = \sigma$$

Radar cross section defined – IEEE standard

- Formally ratio of reflected to incident power density, units m^2

- Rearranged terms in radar equation leads to

$$\sigma = \frac{|E_{scat}|^2}{|E_{inc}|^2} 4\pi R^2$$

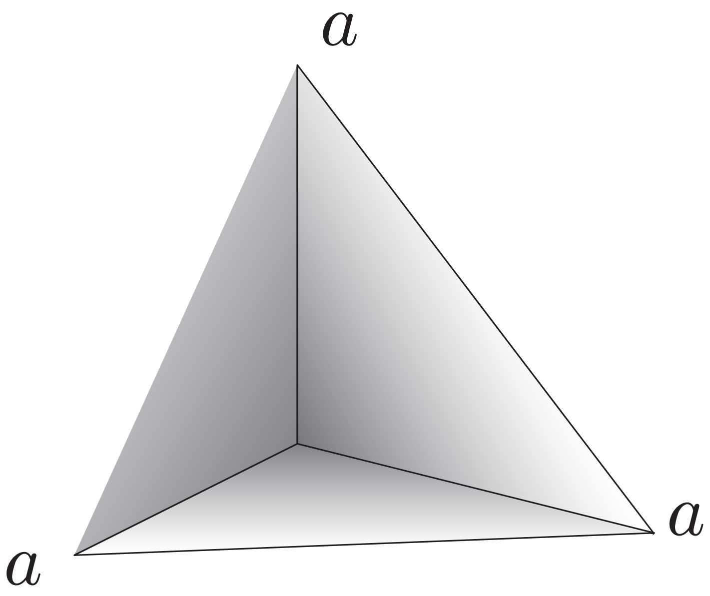


- The last factor accounts for range spreading (dispersion) so that cross section is independent of distance

Radar calibration by using known corner reflector

- Example: RCS (radar cross section) of triangular trihedral corner reflector – *a* is length of side from corner to vertex

$$\sigma = \frac{4}{3}\frac{\pi a^4}{\lambda^2}$$



Note that this calibrates σ only

RCS of surfaces – apropos imaging systems

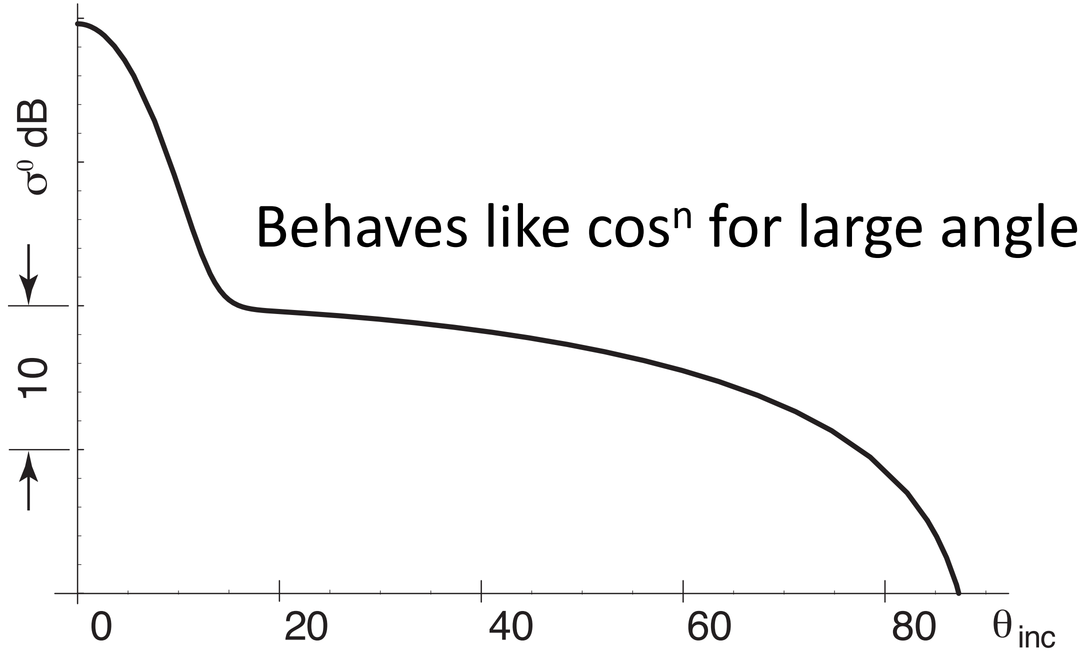
- Traditional approach is to define a normalized radar cross section σ^0
- σ^0 is cross section σ divided by ground area – "specific cross section"
- Then radar equation becomes

$$P_{signal} = \frac{P_t G_t}{4\pi R^2}\, \sigma^0 A_{surface}\, \frac{A_r}{4\pi R^2}$$

- $A_{surface}$ is ground area illuminated, gives RCS units m^2
- Then unitless σ^0 is independent of resolution and reflects a property of the surface
- ***Note: some systems use local incidence angle and some use ellipsoid angle – ellipsoid is more common in literature, although "wrong"***

Examples of σ^0 behavior

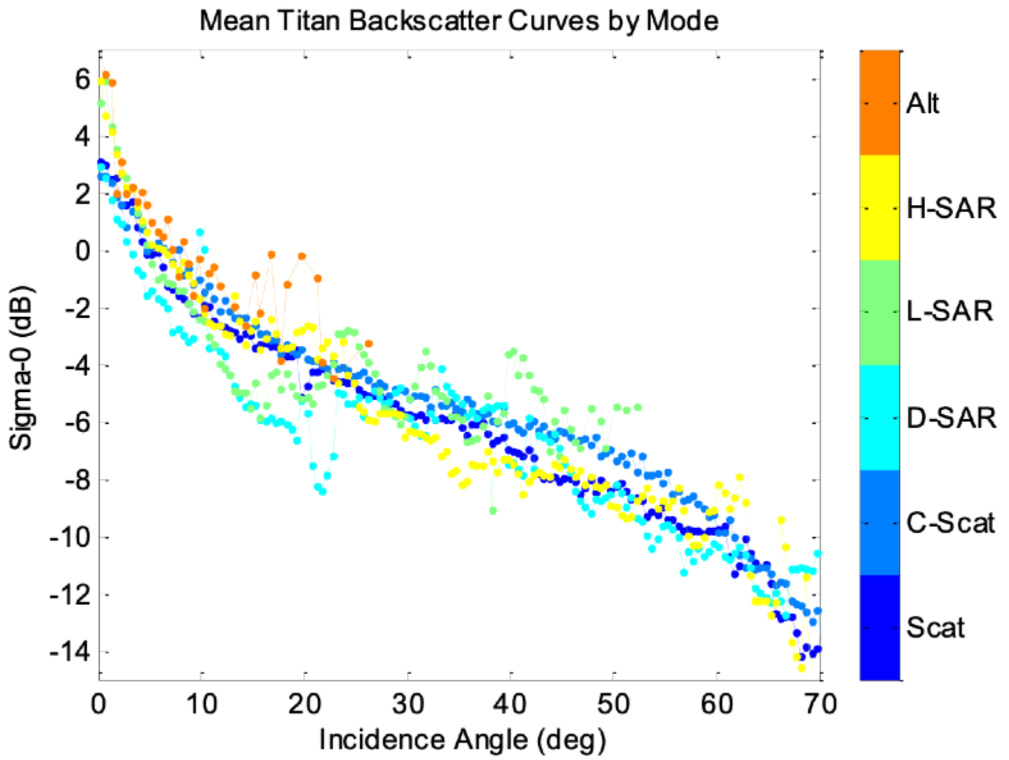
- Generic backscatter model: specular scattering near normal and diffuse scattering for θ > ~15^o

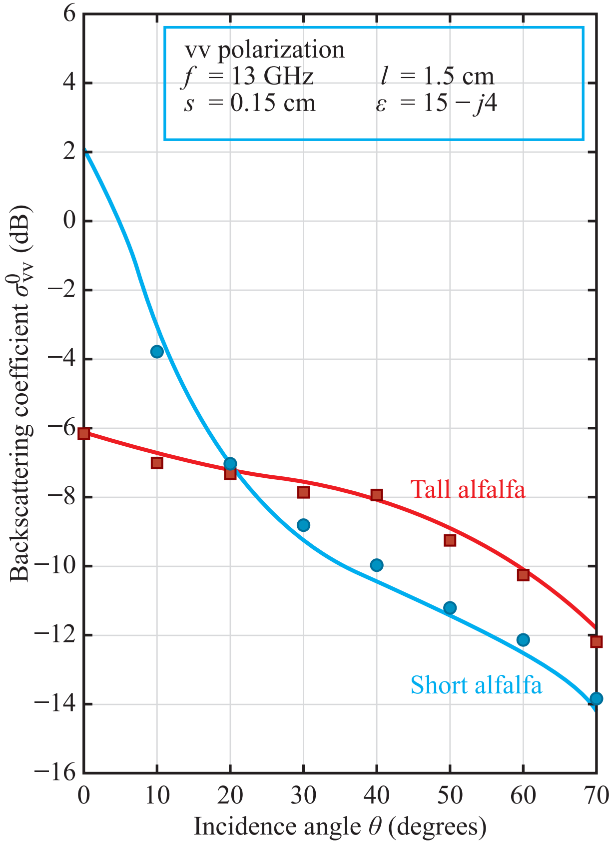


Behaves like $\cos^n$ for large angles

Measured σ^0 behavior

- Same general behavior for many surfaces – on Earth and elsewhere





σ^0 theoretical and empirical relations - $\cos^n$

- Rough surface – Bragg scatter
  - $\sigma^0(\theta) = 16\pi\cos^4\theta\ |g(\theta)|^2\, W(k_B)$
  - g depends on polarization, *W* is surface spectrum at Bragg freq.
- Diffuse (volume) scatter
  - $\sigma^0(\theta) = A\cos^n(\theta)$
  - Isotropic surface n=1, Lambertian surface (e.g. moon) n=2

Normalizing geometrical effects

- σ^0 "corrects" for resolution area dependence on incidence angle
  - Average incidence angle (ellipsoid correction)
  - Local incidence angle (terrain correction)
- NB: local terrain affects scattering area – hence "terrain correction"
- But there is no actual standard (e.g., IEEE) definition of γ^0 or σ^0
  - Rather, most papers default to definitions as used by Small (2011)
  - And this γ^0 includes an extra cosine to compensate for generic falloff as it better flattens the brightnesses
- γ^0 is a better quantity for classification or machine learning approaches – see below

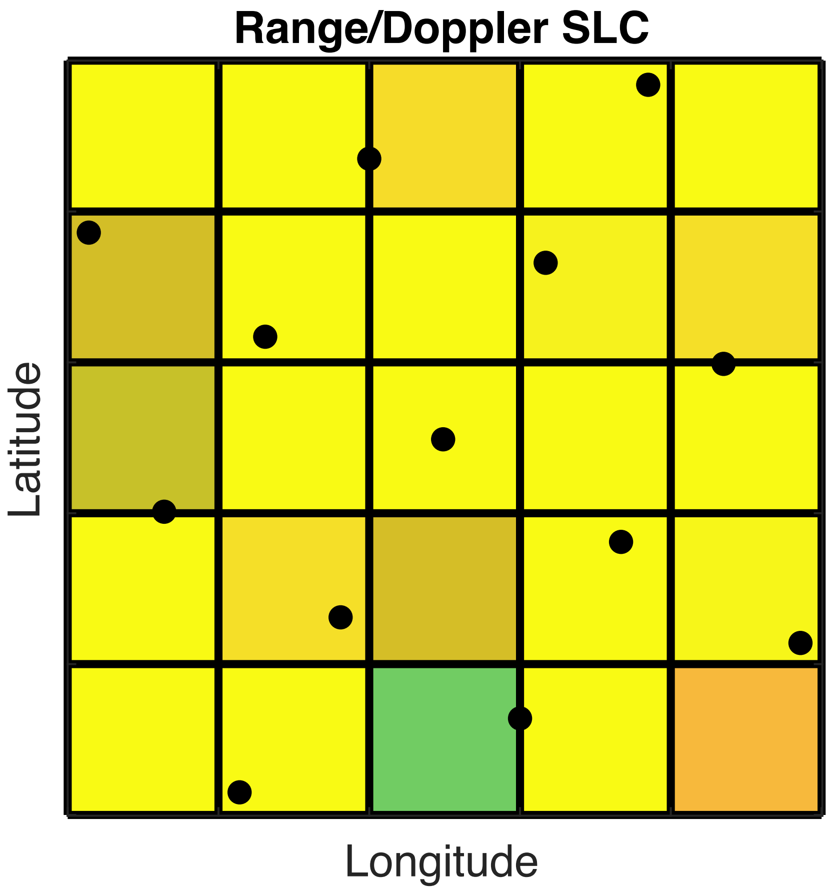
Note on β^0 and γ^0

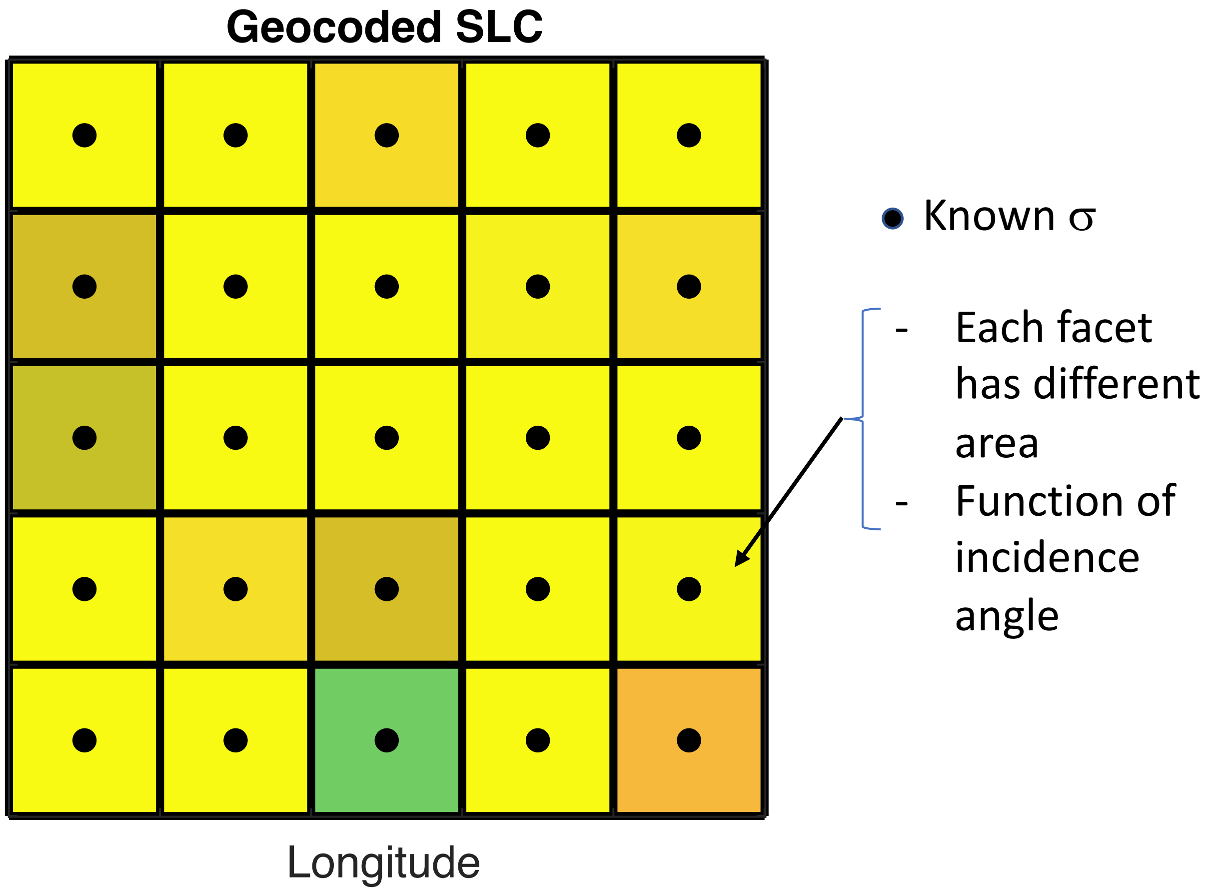
- If normalized cross section is done in range-Doppler plane then instead of σ^0 we have something proportional to cross section itself
- Many authors call this quantity β^0
- Again, no standard definition but would be reducible to either σ^0 or γ^0 if average or local incidence is available as a layer in product
- EXCEPT - γ^0 has the extra cosine in the empirically-derived definition

So why use which of γ^0 or σ^0

- γ^0 decreases sensitivity of cross section to incidence angle, so that variations seen are more dependent on other factors
- This is very helpful for supervised classification
  - Applicable to empirical (allometric) models
  - Machine learning classifiers benefit from removing one degree of freedom
- σ^0 is closer to physical scattering models so better explains backscatter mechanisms
  - Needed for model development
  - Matches EM literature

Scattering area and inc. angle: RSLC vs GSLC





Ease of implementing in geocoded products

- Given σ as calibrated from corner reflectors:
  - $\beta^0 = \sigma/(\Delta r \bullet \Delta az)$ (pixel size in *range/Doppler domain*)
  - $\beta^0 = \sigma/(\Delta x \bullet \Delta y)$ (pixel size in *geocoded ground coords*)
- Areas can be tricky to match in range-Doppler space when data finally used for real applications

- In geocoded domain geometry is clear:
  - $\sigma^0 = \beta^0 \sin \theta_{loc}$ (σ^0 with incidence angle from local normal)
  - $\gamma^0 = \beta^0 \sin \theta_{loc} / \cos \theta_{loc}$ (scattering behavior compensation)

Pros/cons of various options

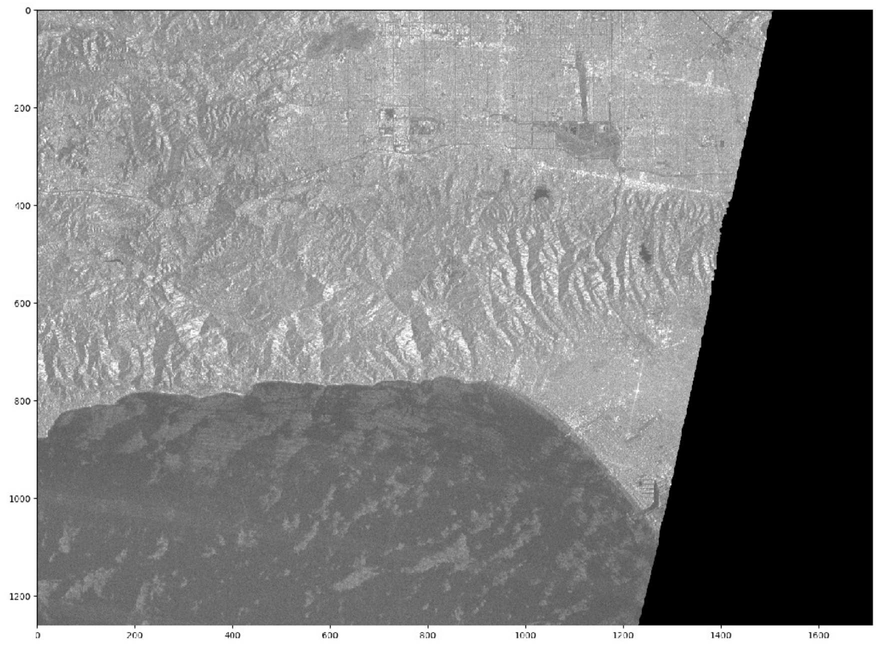
- Both range-Doppler/Geocoded complex images (RSLCs/GSLCs) can be calibrated so that squaring the amplitudes gives units of one of the three quantities – seems needed at minimum
- Geocoded covariance products could be in any of the three as well
- σ^0 is the traditional and accepted quantity and best suited for comparing results to scattering models
- γ^0 gives a flatter image, making time series easier to interpret if multiple incidence angles are used, and emphasizes scattering mechanism changes
- To decide between σ^0 or γ^0 : reduce to scattering-driven measurements or one that adds in an extra cosine for classification/empirical parameters

Example images – these are from Sentinel-1

- Derived from either L0 or L1 data products – both implemented in our codes
- Each panel shows three images: upper right is σ (cross section), lower left is σ^0 (normalized cross section), lower right is γ^0 (extra cosine applied)
- Examples are from LA, Vancouver BC, New England, and Colorado
- Individual swaths still lack an overall scale factor in L0 products, these variations don't appear in calibrated L1 products. See image example at end for this case over LA.

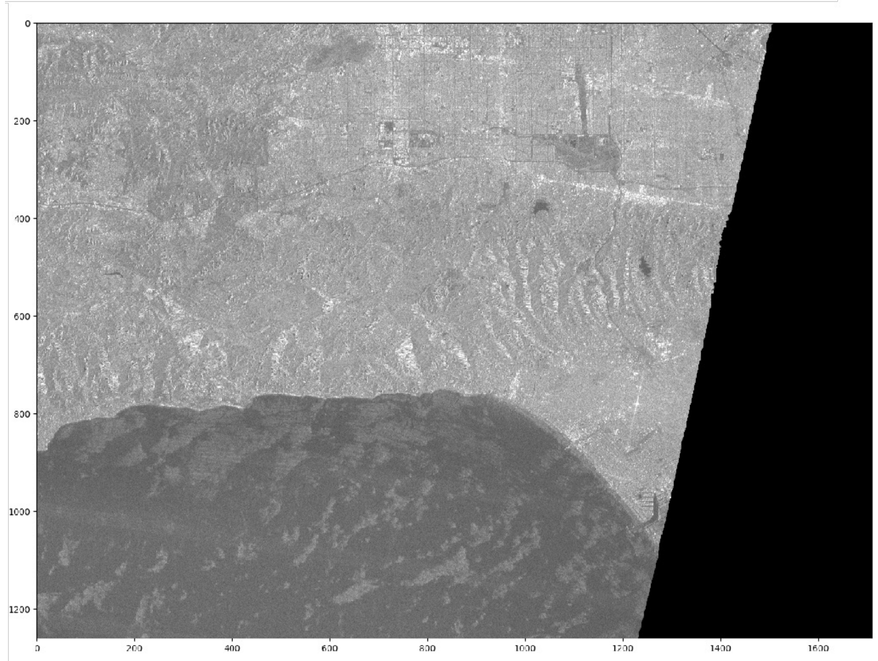
LA - west

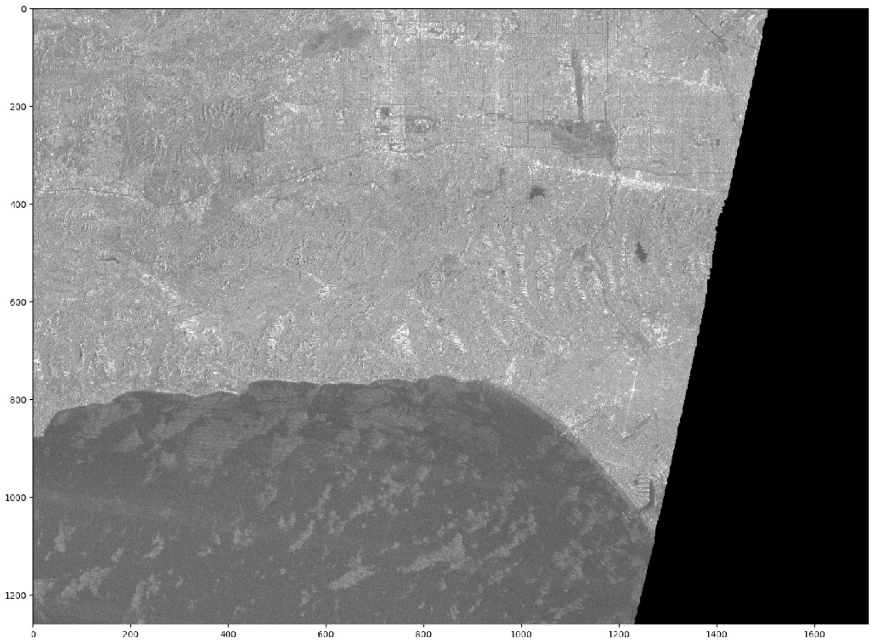
σ



σ^0

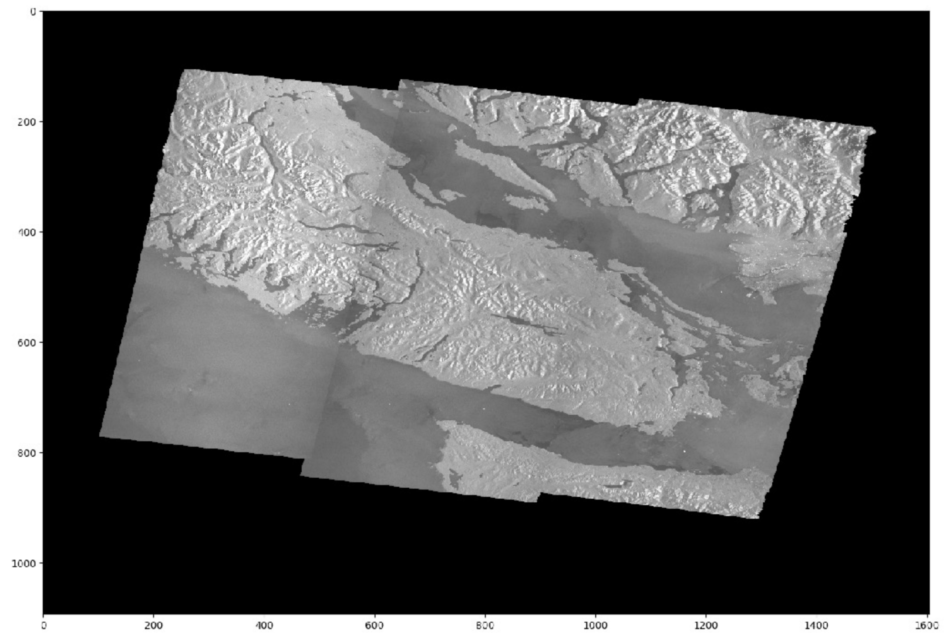
γ^0



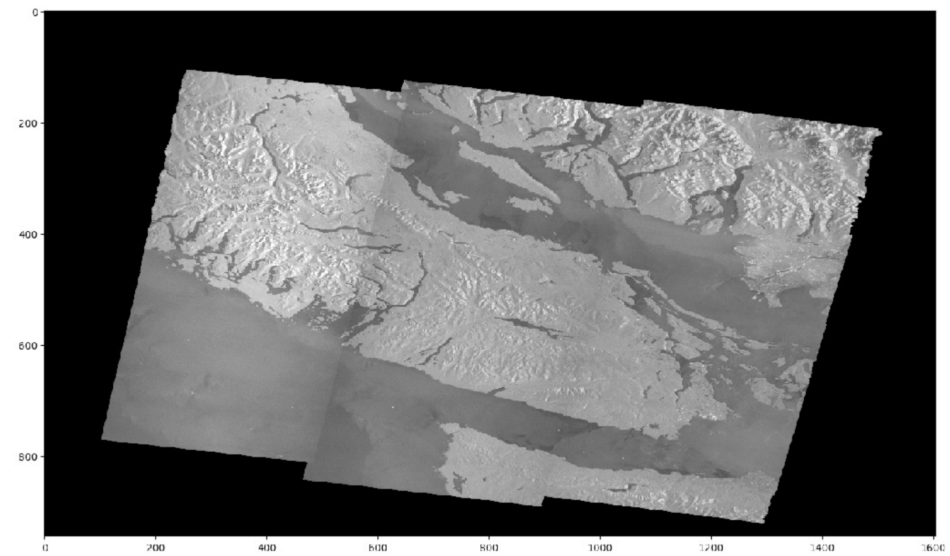


Vancouver BC

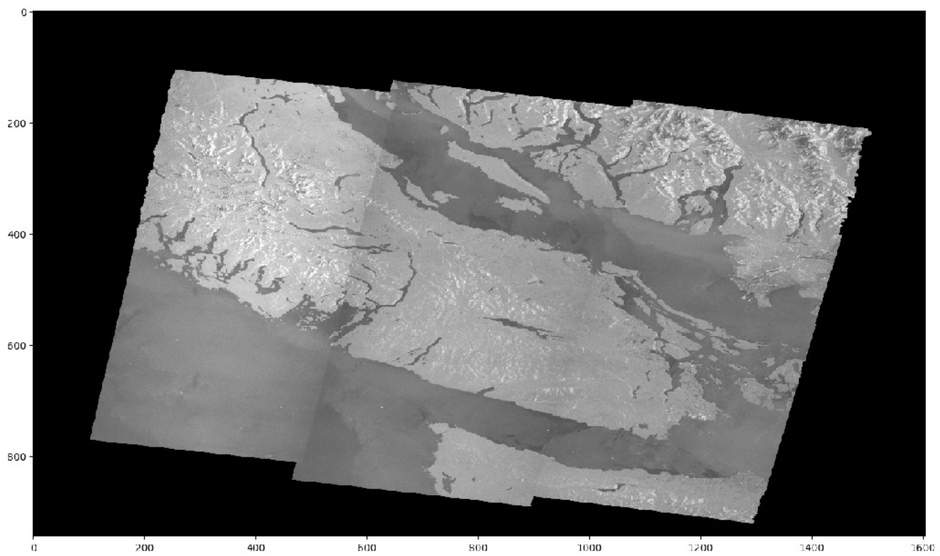
σ



σ^0

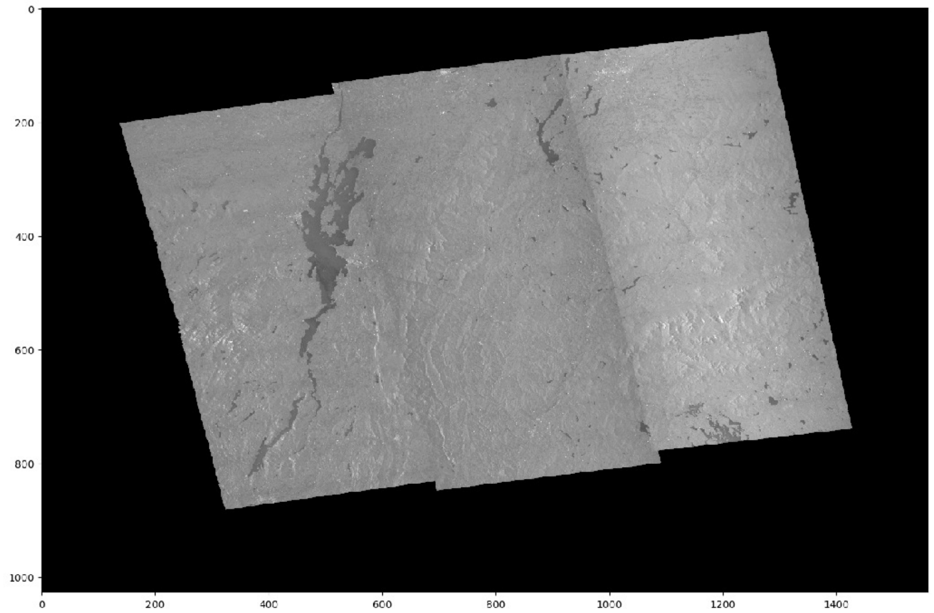


γ^0

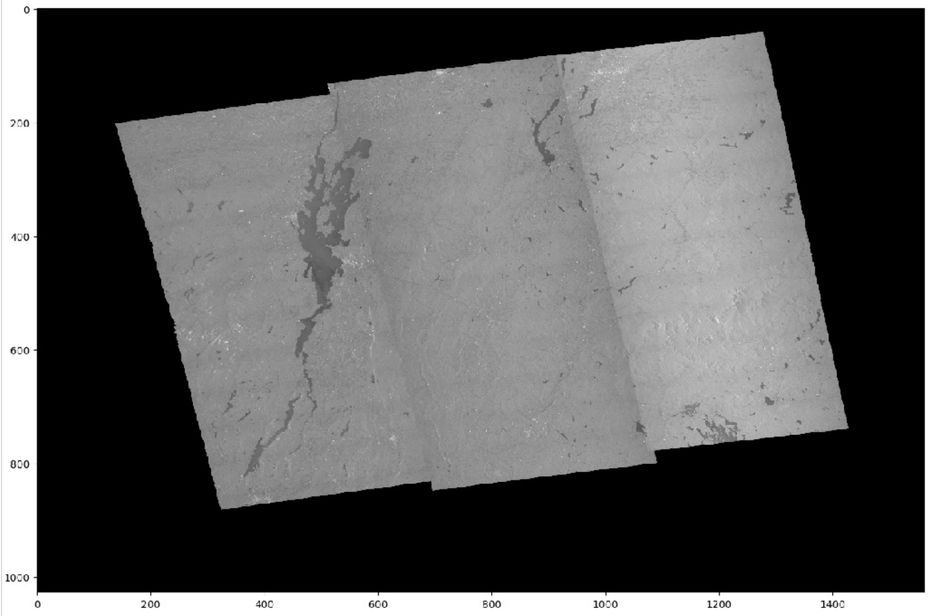


New England

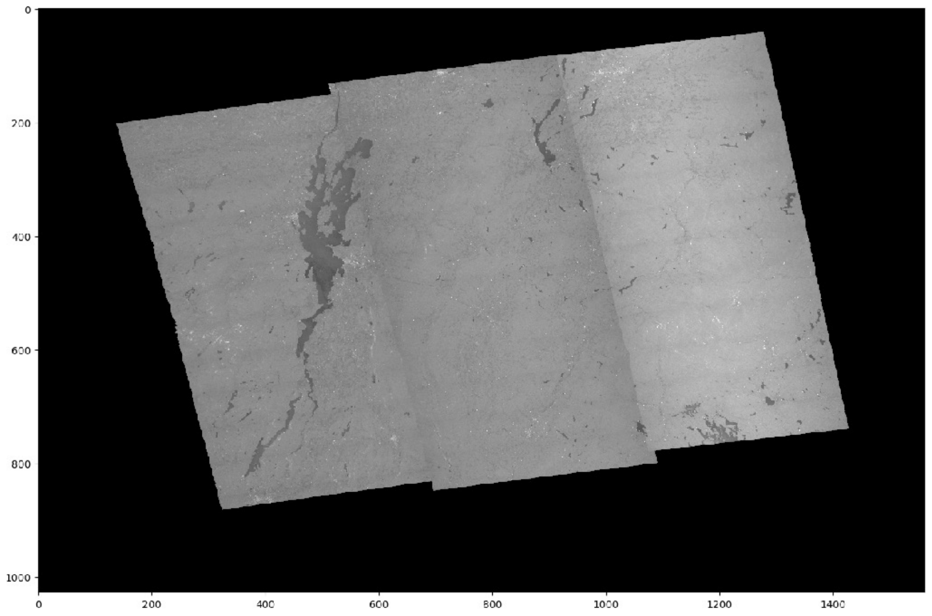
σ



σ^0

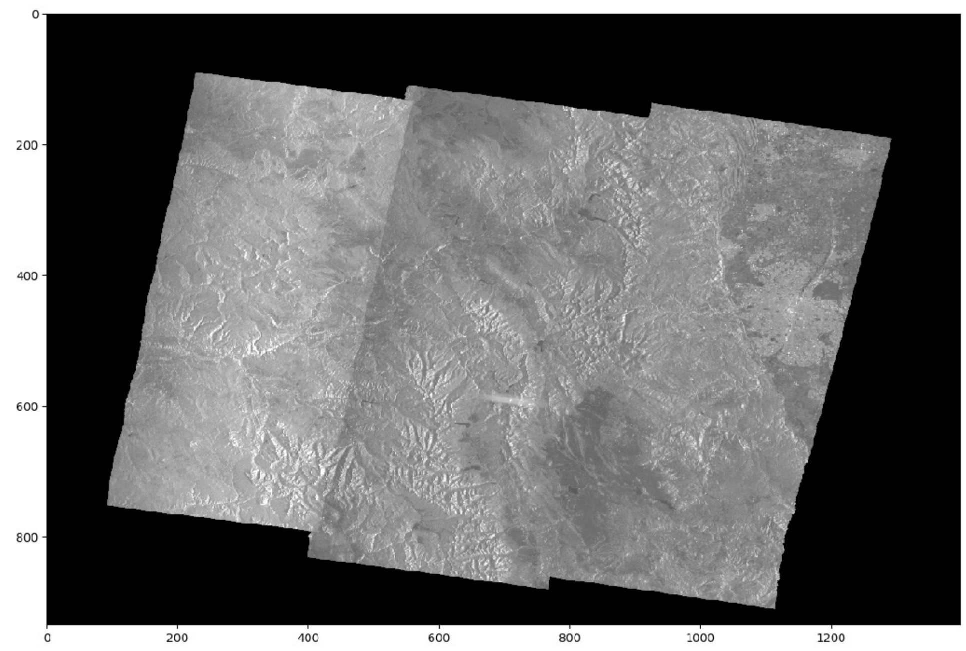


γ^0

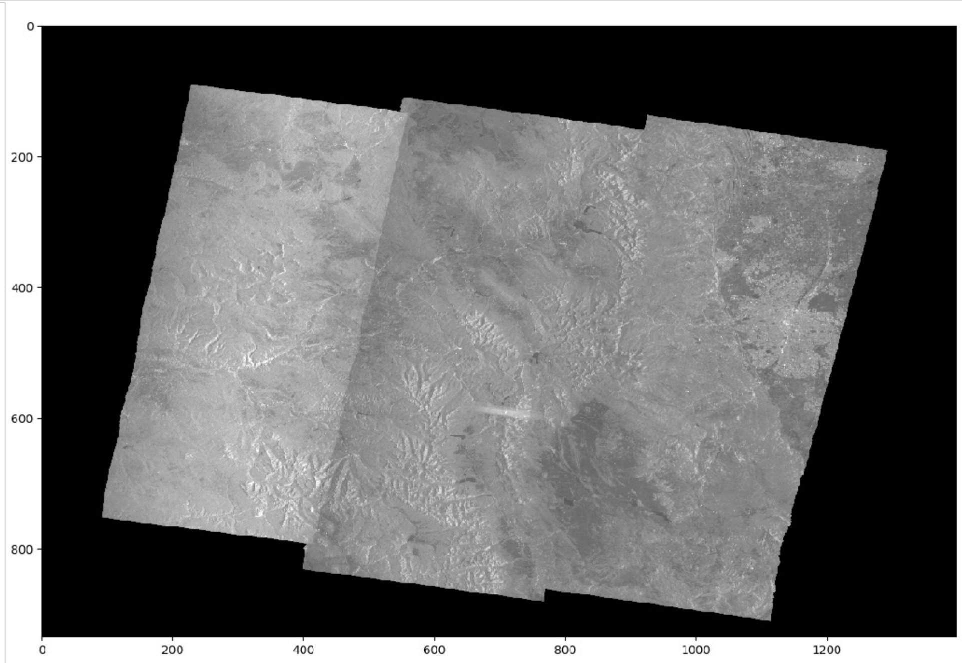


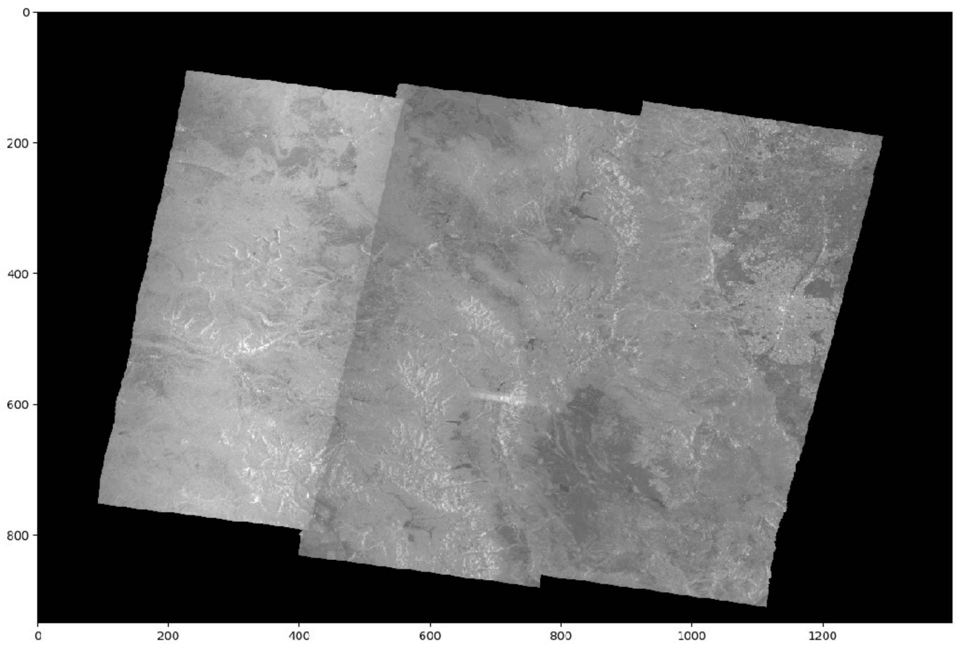
Colorado

σ



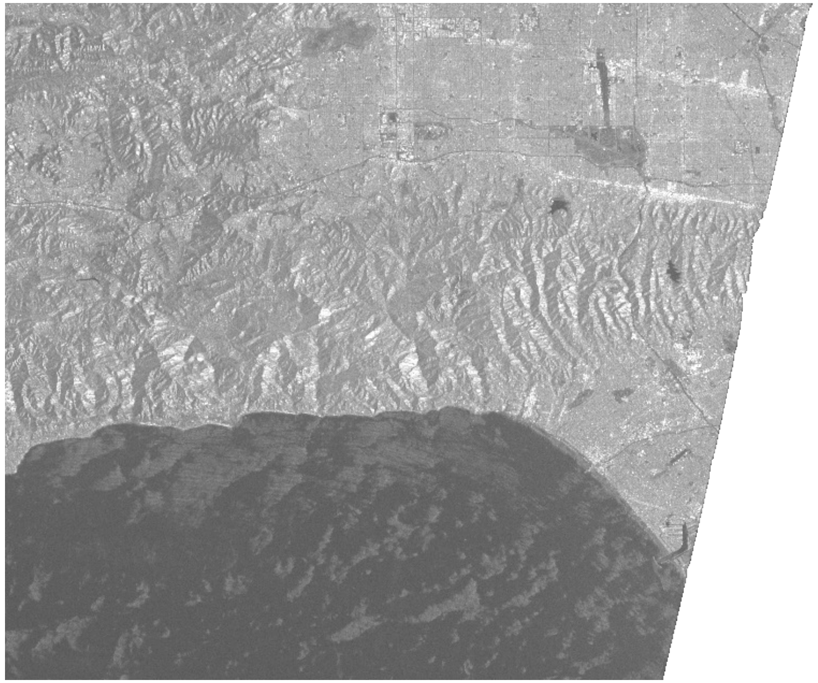
σ^0 γ^0



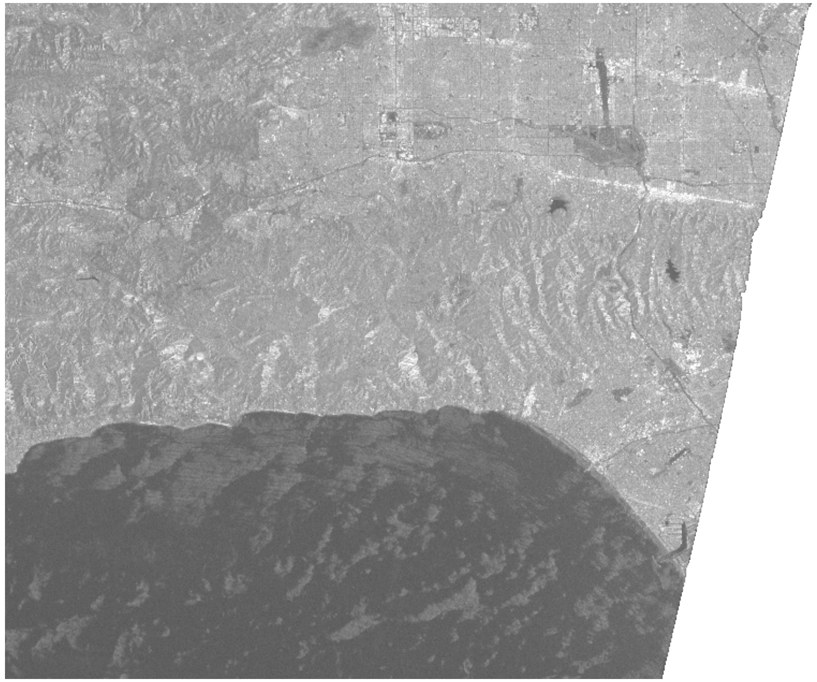


LA derived from calibrated L1 products – more uniform amplitudes

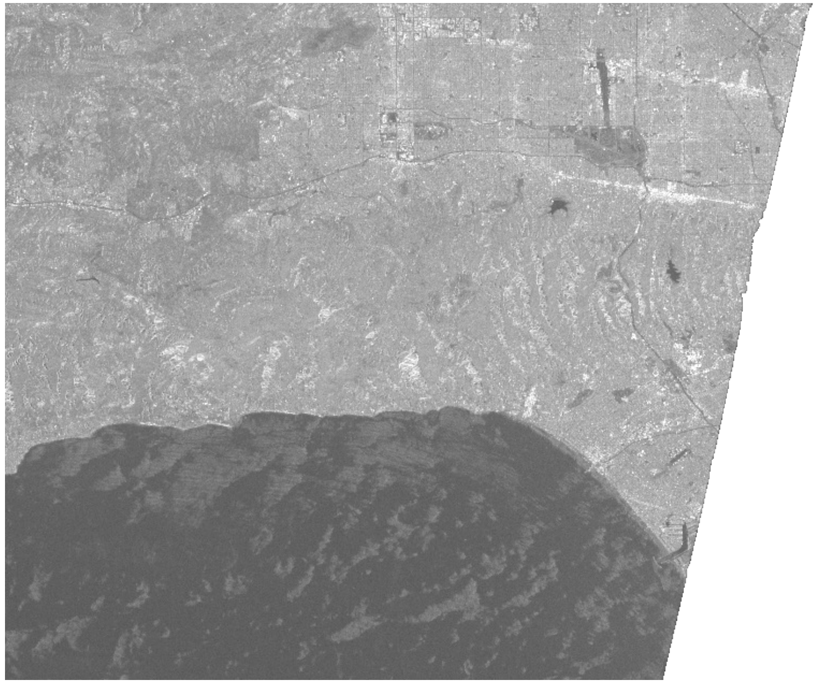
σ



σ^0



γ^0



Summary: Questions for CEOS product generation

- Which of these quantities do we want?
- Do products allow for converting from one to another?
- Do product specs clearly identify which quantities are presented in project products?

Lessons for SAR calibration/validation

- Create RSLCs/GSLCs such that squaring the amplitudes gives units of one of the three quantities – be explicit about units/definitions
- Include local terrain slope or area so that the three quantities β^0, σ^0 or γ^0 may be readily computed regardless of which is presented directly
- Products in geocoded coordinates make the conversions simplest
- Can include DEM itself but that requires user to derive slopes

Reference: some relations between σ^0, β^0 and γ^0 from literature

- Given σ as calibrated from corner reflectors:
- $\beta^0 = \sigma/(\Delta r \bullet \Delta az)$ (pixel size in *range/Doppler domain*)
- $\beta^0 = \sigma/(\Delta x \bullet \Delta y)$ (pixel size in *geocoded ground coords*)

Below apropos working in *geocoded image coordinates*

- $\sigma^0 = \beta^0 \sin \theta_{ell}$ (incidence angle from ellipsoid normal, usual def., ground coords)
- σ^0 or $\gamma^0 = \beta^0 \sin \theta_{loc}$ (σ^0 with incidence angle from local normal, or D. Small $\sigma^0{}_t$)
- $\gamma^0 = \beta^0 \sin \theta_{ell}/\cos \theta_{loc}$ (another possibility for γ^0)
- $\gamma^0 = \beta^0 \sin \theta_{loc} /\cos \theta_{loc}$ (this is a more consistent definition)