Imaging Radar
EE355/GP265
HW 4 Solutions

Question 1:

a) How many valid range bins are found in the range compressed data?

The number of complex range samples is:

$$n_{echo} = \frac{10218-412}{2} = 4903 \text{ samples}$$

The number of samples in the chirp is:

$$n_{chirp} = \lfloor \tau f_s \rfloor = \lfloor (37.12\mu s)(18.96\ MHz) \rfloor = 703 \text{ samples}$$

So, the number of valid range bins is:

$$n_{valid} = n_{echo} - n_{chirp} = 4903 - 703 = 4200 \text{ valid range bins}$$

b) What is the minimum fft size for range processing?

The minimum fft size for range processing is the same as the length of the original echo, which is 4903 samples. If we want this to be a power of 2, I can zero pad up to 8192 samples, keeping in mind that I should only keep the valid data after compressions (4200). With our small data set, zero-padding should not be necessary.

c) What is the effective spacecraft velocity?

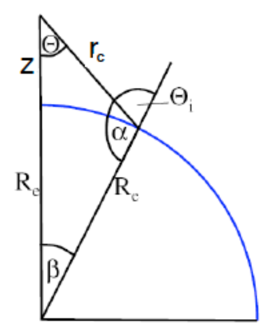

Figure 1: Spherical earth geometry.

The range to the swath center r_c is:

$$r_c = r_0 + \Delta r \frac{n_{valid}}{2} = r_0 + \frac{c}{2f_s}\frac{n_{valid}}{2} = 83000\ m + \frac{(2.99792458e8 \frac{m}{s})}{2(18.96e6\ Hz)}\frac{4200}{2} = 846602.43 \text{ m}$$

We can solve for the platform altitude z, given the spherical earth geometry in Figure 1. From the law of cosines, I have (θ is the look angle to swatch center, R_e is the radius of the earth, r_c is the range to the center of the swatch):

$$\cos\theta = \frac{r_c^2 + (z + R_e)^2 - R_e^2}{2r_c(z + R_e)}$$

$$\ggg z = r_c \cos\theta - R_e + \sqrt{R_e^2 - r_c^2 (\sin\theta)^2}$$

Plugging in $\theta = 23^{\circ}$, R_e = 6378000 m, and r_c = 846602.43 m, I get z = 770717.58 m.

Then, the effective spacecraft velocity is (v = 7550 m/s as the platform velocity):

$$v_{eff} = v\sqrt{\frac{R_e}{z + R_e}} = \left(7550\,\frac{m}{s}\right)\sqrt{\frac{6378000}{770717.58+6378000}} = 7131.41 \text{ m/s}$$

d) What is the range bin spacing in meters?

The slant range bin spacing Δr is:

$$\Delta r = \frac{c}{2f_s} = \frac{2.99792458e8\ m/s}{2(18.96e6\ Hz)} = 7.91 \text{ m/pixel}$$

The slant range resolution δr is:

$$\delta r = \frac{c}{2(BW)} = \frac{c}{2s\tau} = \frac{2.99792458e8\ m/s}{2(4.189166e11\frac{Hz}{s})(37.12e{-6}\ s)} = 9.64 \text{ m}$$

To get the ground range resolution and bin spacing, we first need the incidence angle θ_i at the center of the swath, which is larger than the original look angel, since we have a spherical earth geometry. By using the law of sines (here, β is the angle at the center of the earth):

$$\frac{\sin\beta}{r_c} = \frac{\sin\theta}{R_e} ==> \beta = \sin^{-1}\left(\frac{r_c}{R_e}\sin\theta\right)$$

$$\theta_i = \theta + \beta = 23^{\circ} + 2.973^{\circ} = 25.973^{\circ}$$

The ground range bin spacing is then:

$$\Delta r_g = \frac{\Delta r}{\sin\theta_i} = \frac{7.91\ m/pixel}{\sin 25.973^{\circ}} = 18.05 \text{ m/pixel}$$

And the ground range resolution δr_g is then:

$$\delta r_g = \frac{\delta r}{\sin\theta_i} = \frac{9.64\ m}{\sin 25.973^{\circ}} = 22.01 \text{ m}$$