Homework 1 Solutions

Problem 1. Consider a bistatic configuration where the transmitter and receiver are separated by a distance of 5 km. The radar is operating in the CW interference mode at a wavelength of 5 m. A ship reflects the radar waves as with the 1922 radar of Taylor and Young of NRL. Suppose the ship travels along a perpendicular bisector of the line connecting the transmitter and receiver at a constant velocity of 5 m/s. What will the envelope of the received waveform look like? What will be the period of the interference envelope as a function of ship distance from the line between radar transmitter and receiver? You may take the radar transmitter and receiver and the ship to be in the same plane. Doppler effects can be ignored for the purposes of this problem (think about whether this really matters). Assume that the reflected signal arrives at the receiver with half the magnitude of the direct signal.

Solution. Let $a = 5$ km denote the separation between the transmitter and receiver and let d denote the distance of the ship from the line between the transmitter and receiver as in Fig. 1. The direct signal arriving at the receiver is a time-delayed version of the transmitted

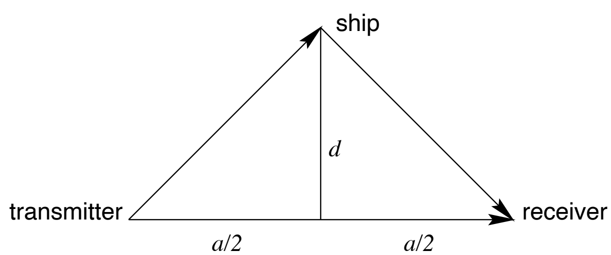

Figure 1: Geometry for problem 1.

sinusoid:

$$s_{\mathrm{dir}}(t) = \cos(2\pi f_0(t - a/c))$$

where c is the speed of light and we assume that the direct signal has unit amplitude. The reflected signal is given by

$$s_{\mathrm{ref}}(t) = \frac{1}{2}\cos\left(2\pi f_0\left(t - \frac{2\sqrt{d^2 + (a/2)^2}}{c}\right)\right)$$

The sum of these signals $s_R(t) = s_{\mathrm{dir}}(t) + s_{\mathrm{ref}}(t)$ is received at the receiver. There are many ways to think about detecting the envelope: a diode detector (as in simple AM demodulation), a mixer and low-pass filter, etc. It is probably easiest to think of the signal in terms of its rotating phasors, however. Fig. 2 shows that the two signals can be represented as vectors in the complex plane, with the envelope being the magnitude of their resultant sum. The envelope will be therefore be given by

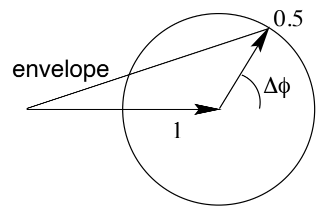

Figure 2: Phasor diagram for direct and reflected signals for problem 1.

$$\sqrt{(1+0.5\cos(\Delta\phi))^2+(0.5\sin(\Delta\phi))^2}=\sqrt{\frac{5}{4}+\cos(\Delta\phi)}$$

This expression can also be approximated to some extent by $1+0.5\cos(\Delta\phi)$. The quantity $\Delta\phi$ is the relative phase difference between the two signals, which varies as a function of distance d. Physically, the direct and reflected signals are sinusoids of the "same" frequency but with different phases, so sometimes the signals interfere constructively while at other times they interfere destructively. Normalized to the magnitude of the direct signal, the envelope will therefore vary between 1.5 and 0.5. By examining $s_{\text{dir}}(t)$ and $s_{\text{ref}}(t)$, we find the phase difference do be

$$\Delta\phi = 2\pi f_0 \frac{a-2\sqrt{d^2+(a/2)^2}}{c}$$

The magnitude of the envelope is plotted as a function of distance in Fig. 3.

Take the period T to be the time between peaks in the envelope. The envelope is not exactly sinusoidal, but its peaks do correspond to the peaks of a sinusoid of the form $\cos(\Delta\phi)$. Using the chain rule and knowing that the time derivative of d is v, the period of a the envelope is given by

$$T=\frac{1}{f_{\text{inst}}}=\frac{2\pi}{\left|\frac{d\Delta\phi}{dt}\right|}=\frac{2\pi}{\left|\frac{d\Delta\phi}{dd}\frac{dd}{dt}\right|}=\frac{\lambda\sqrt{d^2+(a/2)^2}}{2dv}$$

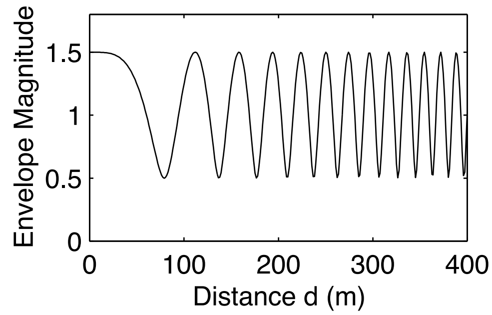

Figure 3: Envelope of received signal for problem 1.

Problem 2. Derive an expression for Doppler shift when there is nonzero target acceleration ($\ddot{R} \neq 0$). (Hint: frequency is the time derivative of phase; consider the phase of the received echo as a function of range.) Neglect relativistic effects.

Solution. Let the transmitted signal and the range at time t be given by

$$v_t(t) = \cos(2\pi f_0 t + \phi_0)$$

$$R(t) = R_0 + vt + at^2/2$$

where v and a are the target velocity and acceleration. The received signal is a time-delayed version of the transmitted signal:

$$v_r(t) = \cos(2\pi f_0 (t - \tau) + \phi_0)$$

with the delay τ given by

$$\tau = \frac{2R(t)}{c}$$

If we assume that the target velocity and acceleration are much, much smaller than the speed of light, the change in range during the signal flight time is insignificant. Substituting, we obtain

$$v_r(t) = \cos\left(2\pi f_0 \left(t - \frac{2}{c}(R_0 + vt + at^2/2)\right)\right)$$

As frequency is the time derivative of phase, we differentiate the argument of the cosine above