

Scattering laws and data analysis

Problem 1. For this problem we will be using a form of the Bragg scattering law as follows:

$$\sigma_0 \text{ (in dB)} = 10 * \log_{10} (8 * k^4 * h^2 * \cos(\theta)^4 * (2 * k * \sin(\theta))^{-\text{slope}})$$

(note negative sign in front of the slope parameter)

where k is the wavenumber ($2 * \pi / \lambda$), h is the surface roughness standard deviation in m, θ is the incidence angle, and slope is the roughness spectral index. Let the wavelength λ be 0.24, same as for NISAR.

- Read the file hw7prob1a.txt. This is a table of incidence angles in degrees followed by σ_0 (in dB). If the roughness spectral index is 2, find the surface rms height h by fitting the data to the model above.
- Next, read hw7prob1b.txt. This is a more restricted set of measurements more consistent with the capabilities of NISAR. Again find the roughness h .
- Now read hw7prob1c.txt. In this example both the spectral index (slope parameter) and the roughness have changed. Find each.

Problem 2. You are using the NASA deep space network to observe the moon at S-band ($\lambda=0.12$). Your other radar parameters are:

Pt = 100 kw

Antenna diameter = 64 m

Moon distance = 384400 km

Moon diameter = 3475 km

- The radar albedo (analogous to σ_0) of an object is defined as the ratio of the radar cross section in m^2 to its projected area from your viewing angle. If you measure a received power of 0.1739 nanowatts (1.739×10^{-10} watts) using the radar parameters listed here, what is the moon's radar albedo?
- We can relate the radar albedo to the product of the Fresnel power reflection coefficient ρ for a known surface dielectric constant and a factor that depends on small scale roughness, as follows:

$$\text{albedo} = \rho \exp(-4 \pi h / \lambda)^2$$

where $\rho = ((1-\sqrt{\epsilon})/(1+\sqrt{\epsilon}))^2$, and ϵ is the dielectric constant of the surface material. The roughness factor can be derived from the Huygens-Fresnel approximation but that is beyond the scope of what we need here. In this equation, ρ is the Fresnel reflection coefficient at normal incidence, h is the roughness standard deviation in m, and λ is the wavelength.

What is the moon surface roughness if the average dielectric constant is 9 (an average value for Earth rocks)?

- c) How about if the surface is dust that is 50% porous instead of solid rock? (Hint: how would the average dielectric constant change?)

*Extra credit: derive the roughness factor assuming that the surface heights are Gaussian distributed with rms height h .