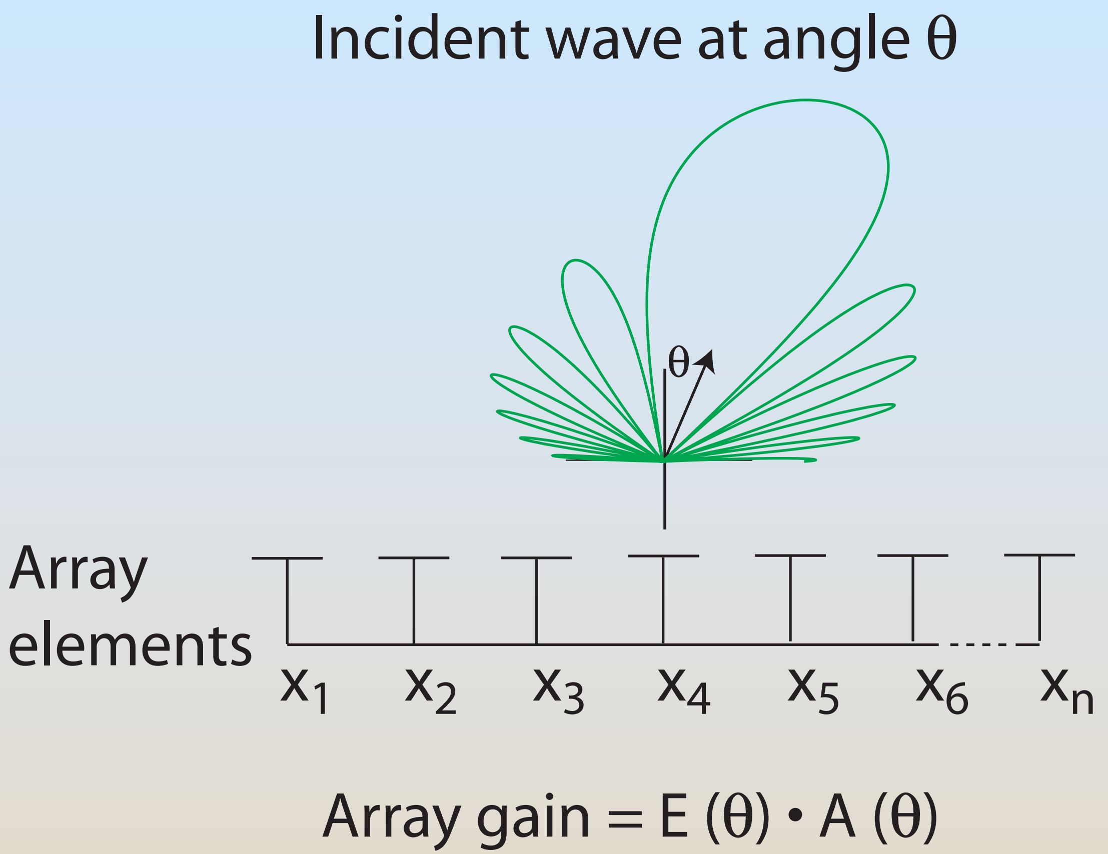
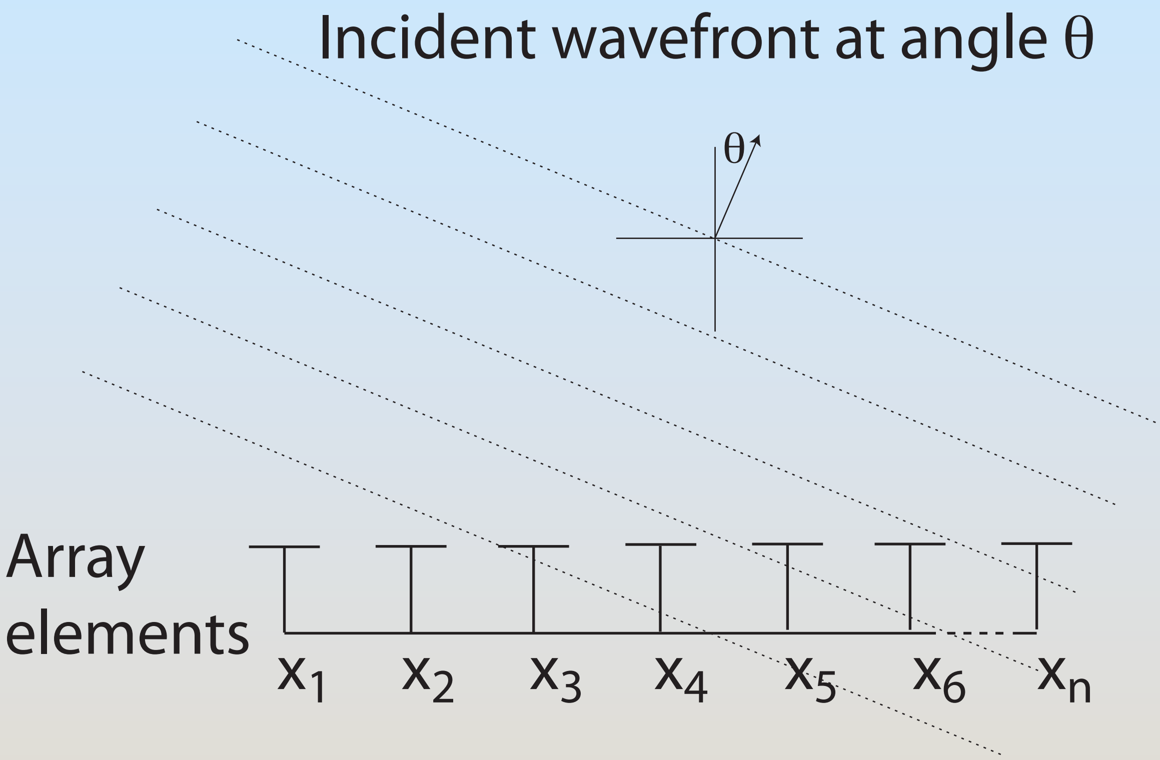
Beamforming in space – beam viewpoint



- Measuring power at each angle images sources in space

Beamforming in space – phase viewpoint

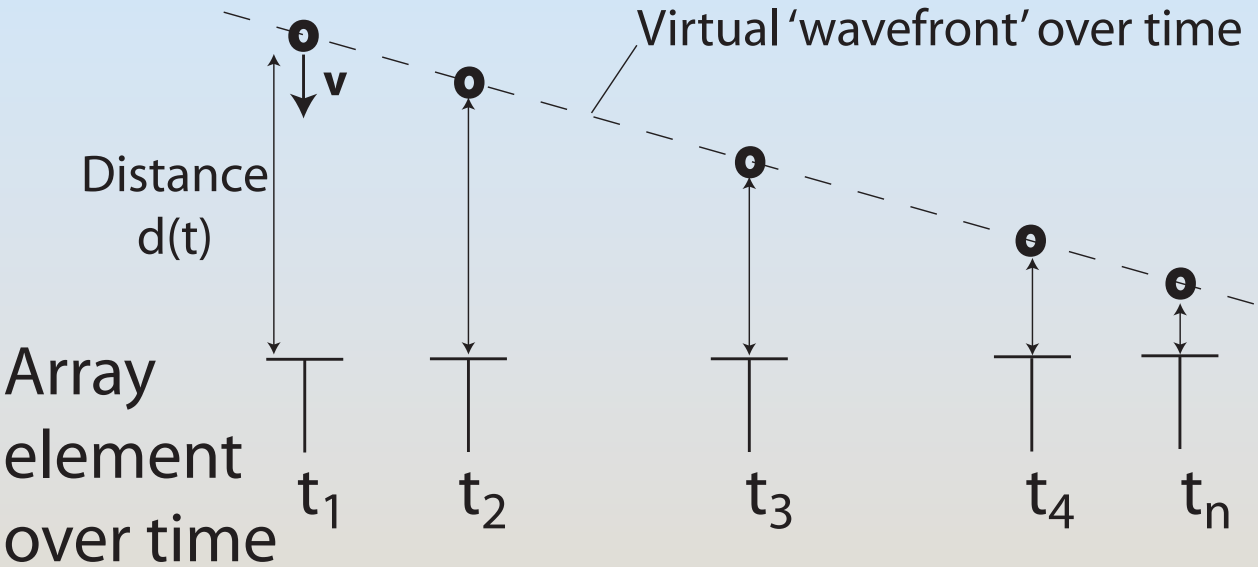


$$\text{Phase at kth element} = \frac{4\pi}{\lambda}\sin\theta\,(x_k - x_{ref})$$

- Measuring phase at each angle finds sources in space

Beamforming in time

Moving object at velocity v

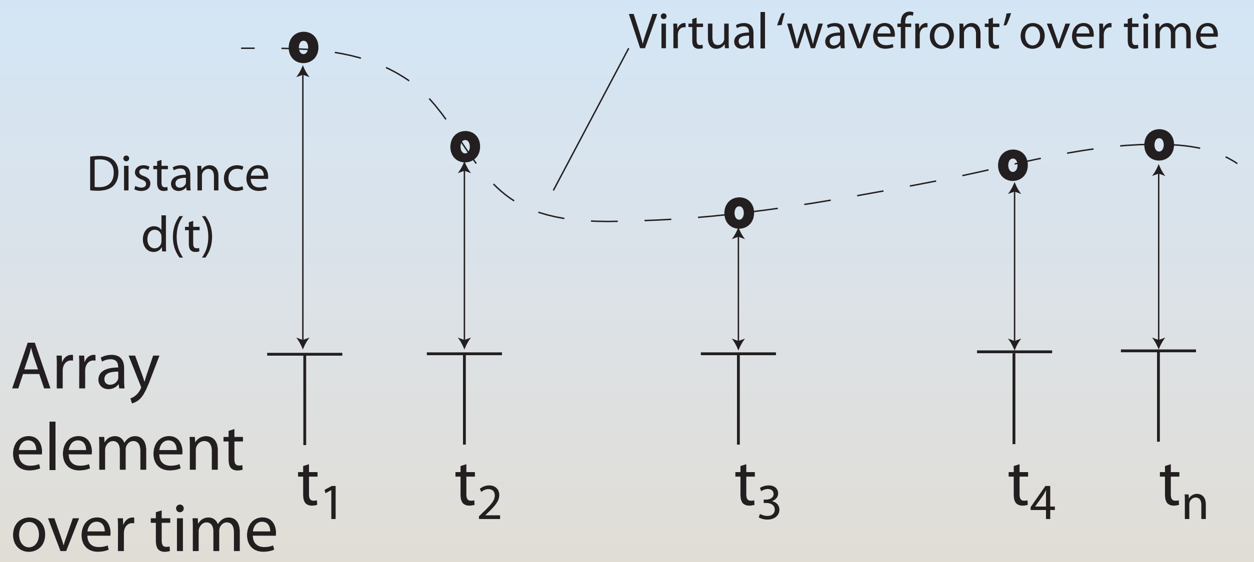


$$\text{Phase at kth time} = \frac{4\pi}{\lambda} d(t_k) = \frac{4\pi}{\lambda} v \cdot t_k$$

- Measuring phase at each time finds line of sight velocity

Time-dependent velocities

Time dependent velocity v(t)

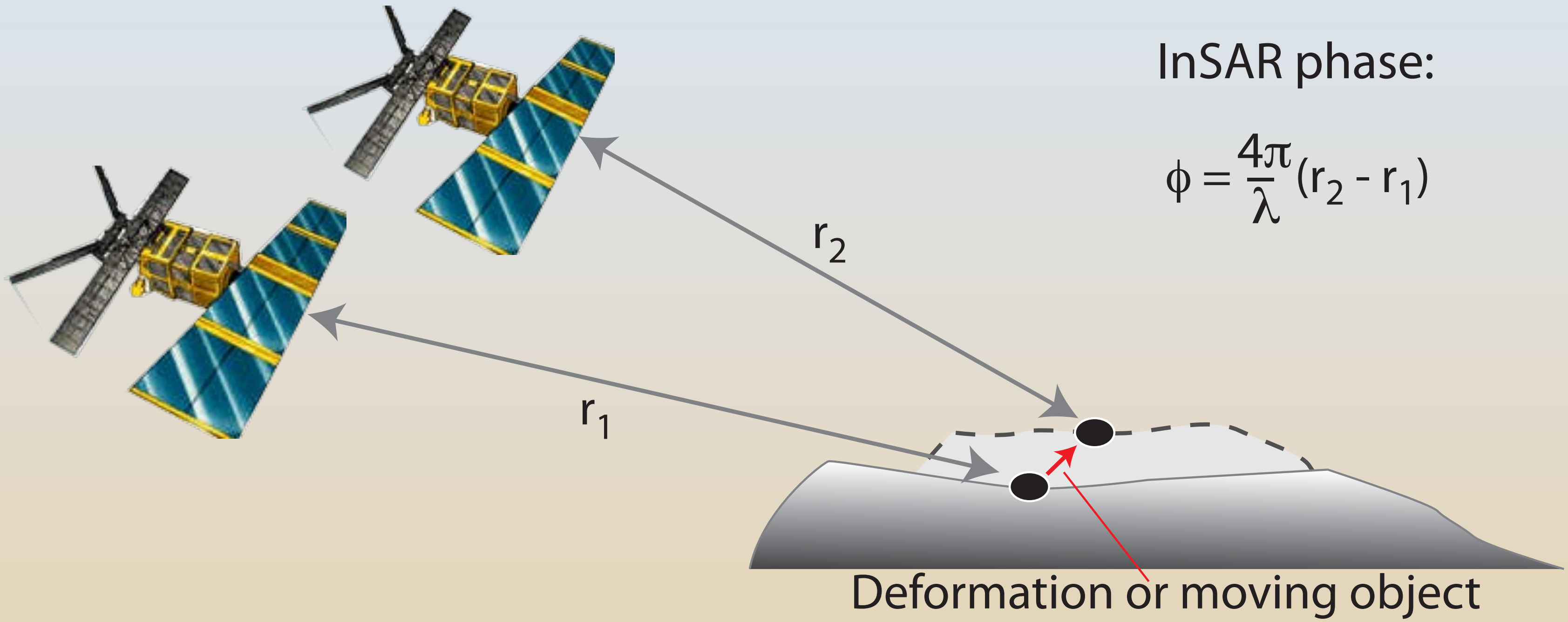


$$\text{Phase at kth time} = \frac{4\pi}{\lambda} d(t_k) = \frac{4\pi}{\lambda} \int_0^{t_k} v(t)\, dt$$

- Measuring phase at each time finds line of sight velocity

The fundamental InSAR measurement

- Observe the same region twice, measure mutual coherence of each resolution element
- Phase difference of radar echoes yields change in path length

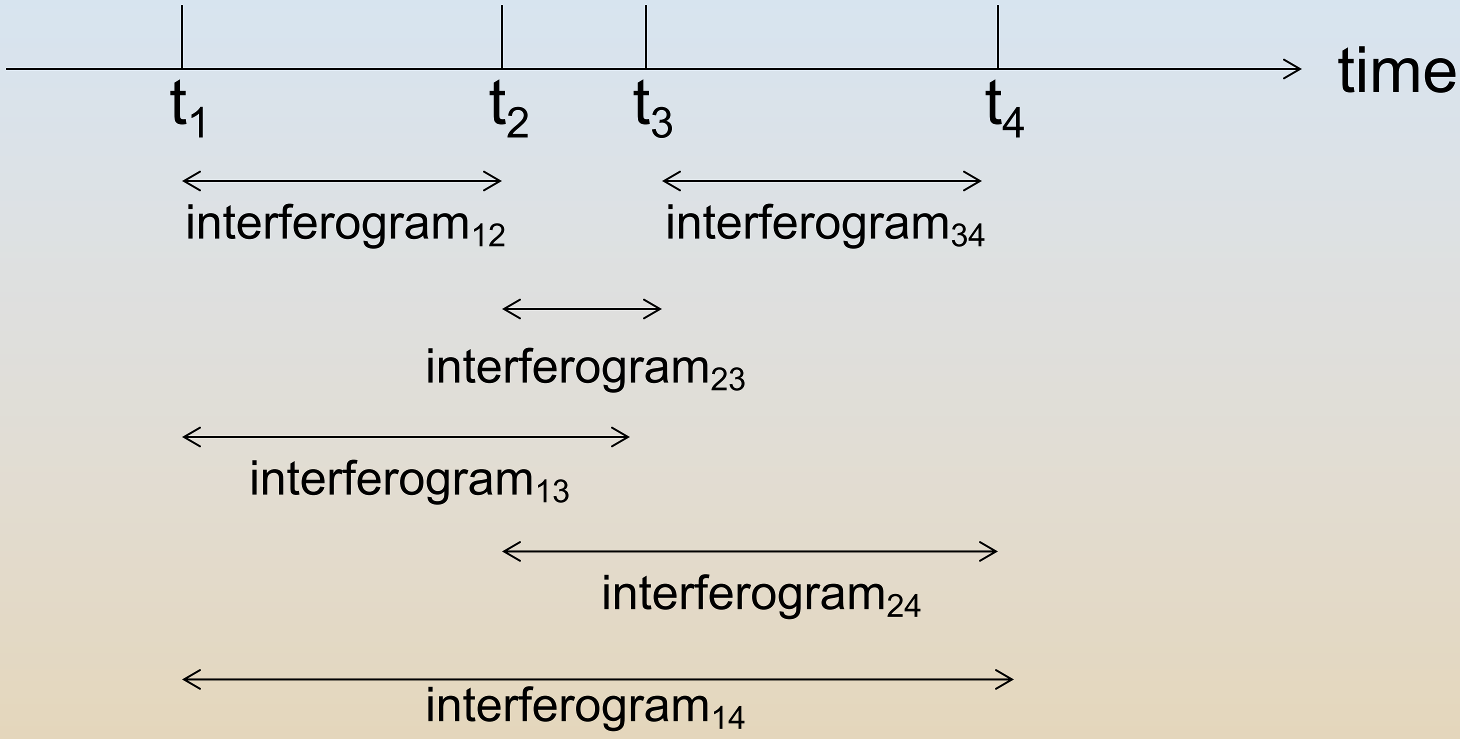


Time series InSAR method

- Acquire many scenes from a ground area
- Precisely coregister the complex image data, form all pairs
- Compute phase differences to find change in range for each pixel
- Extract time-variable velocity for each time interval
- Least squares analysis of overdetermined system

Estimate the velocity series

- Create a set of interferograms for various time intervals
- Suppose we have a series of temporal observations:



Least-squares velocity estimation

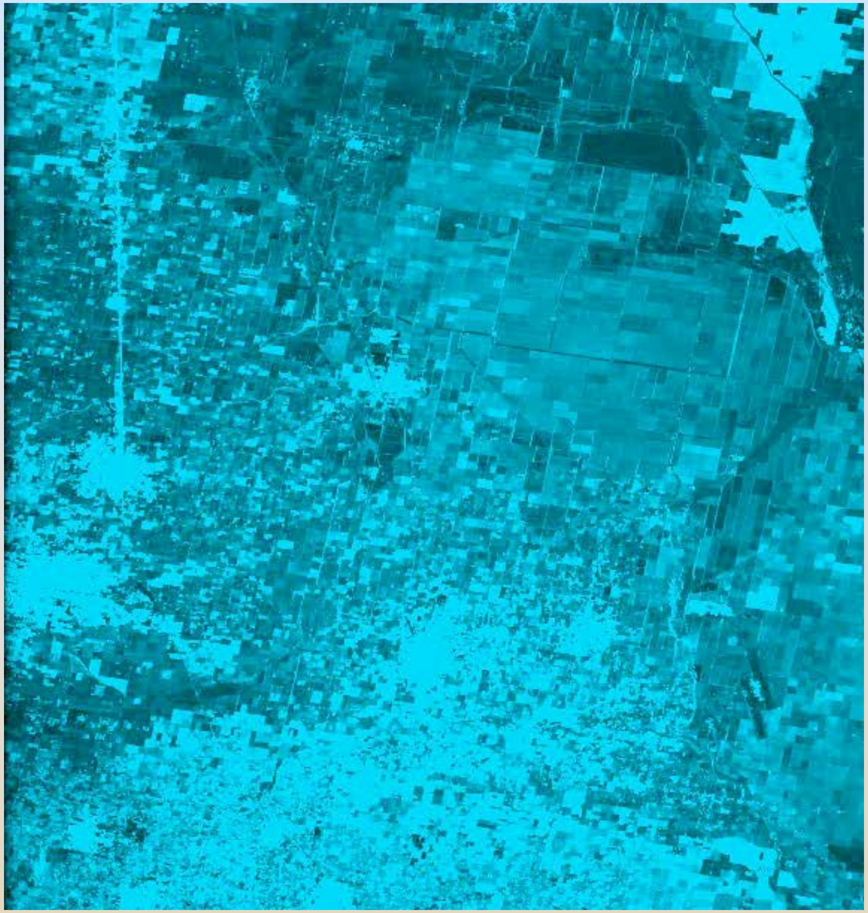
- We derive a series of velocities v at each image point from a set of interferogram measurements $\Delta\phi_{ij}$
- Define a matrix A of time separations describing the full set of interferograms, to obtain a matrix equation

$$Av=\Delta\phi$$

whose solution is the desired time series

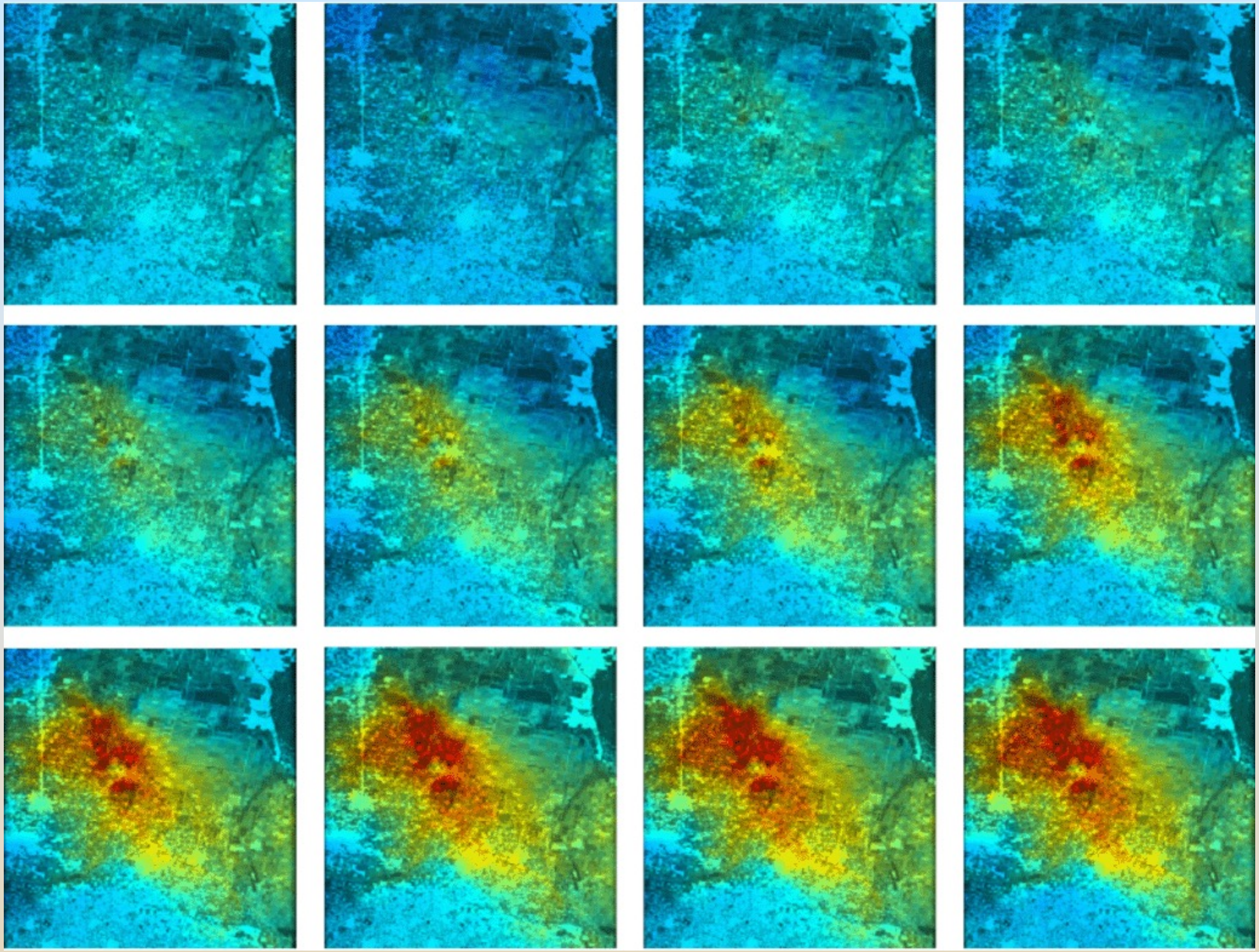
- Repeating this for every pixel in an image, and integrating the velocity series, results in an image of pixel position vs. time

Time series InSAR example:
Kings County groundwater basin



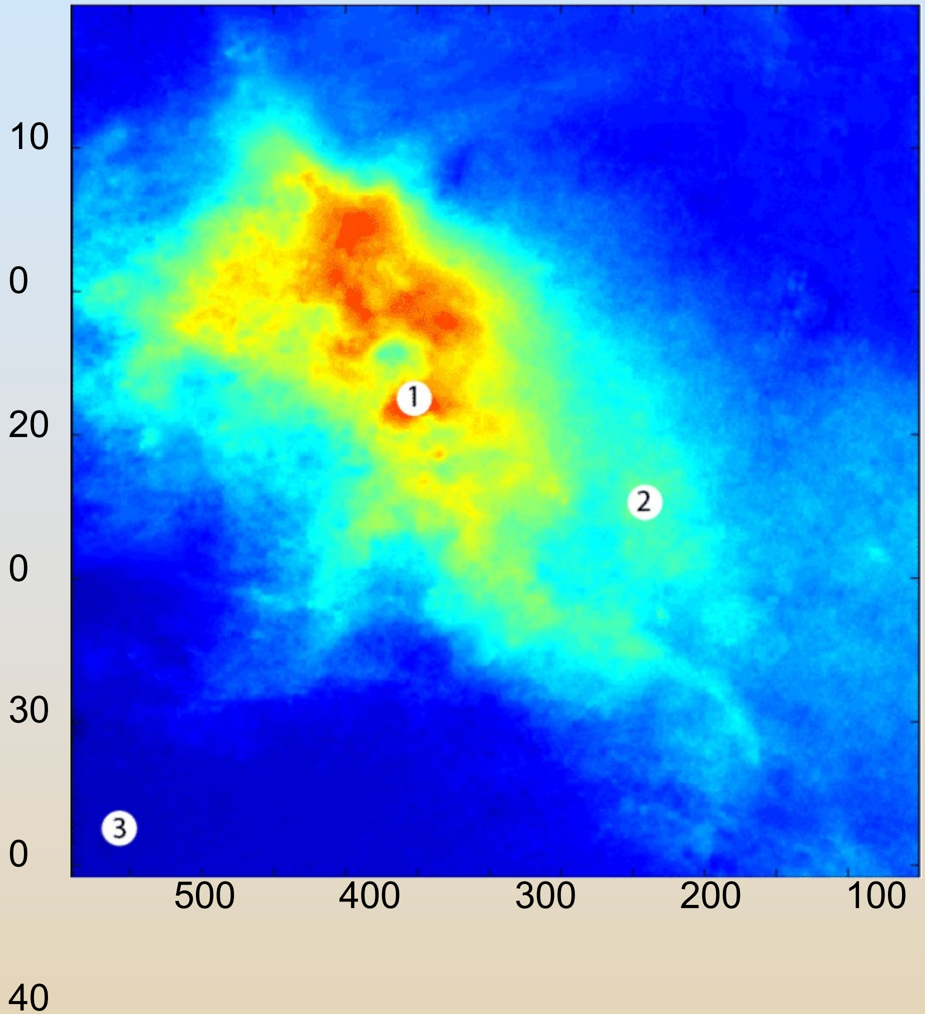
Kings County, CA Groundwater Basin

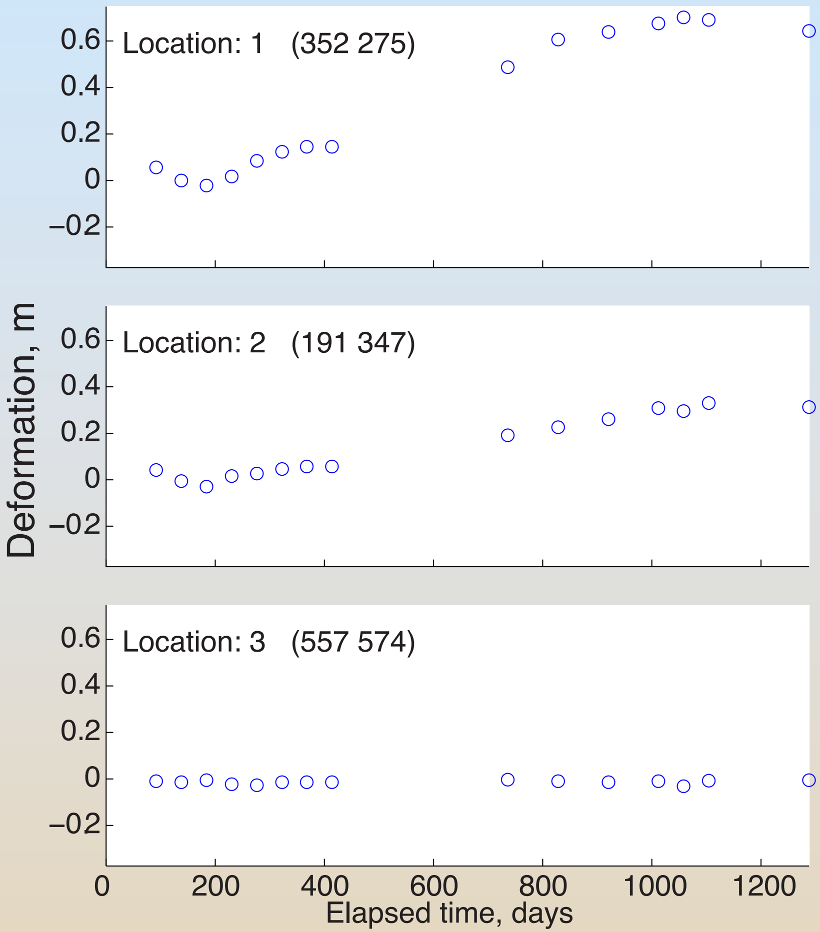
June,
2007



Dec,
2010

Kings County time series





Why is this challenging?

- Coherence decreases with both spatial and temporal satellite separation
- Not all interferograms have sufficient coherence for reliable phase measurements at all pixels
- Need to select either 1) high coherence interferograms or 2) high coherence pixels

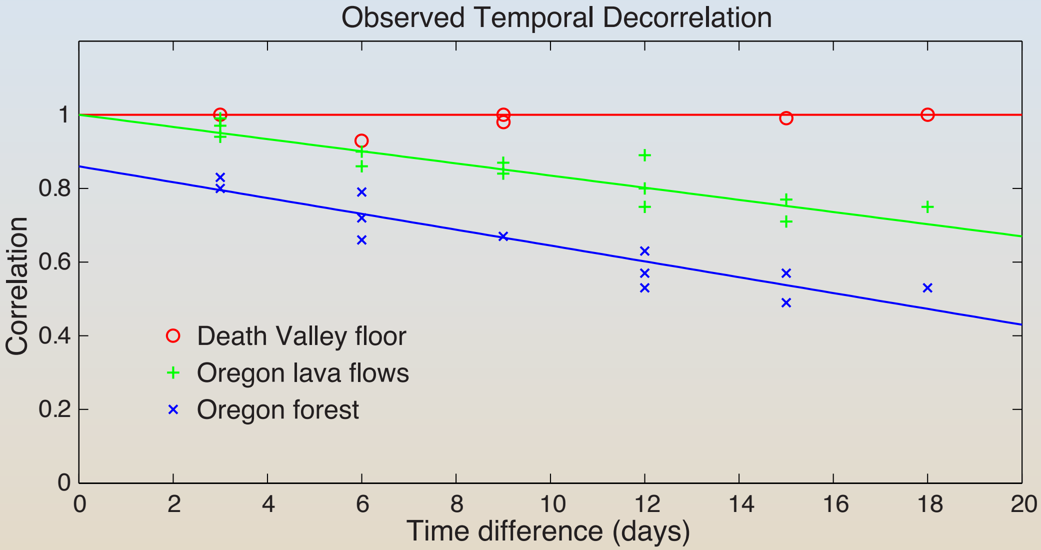
Spatial coherence

- Spatial coherence, or correlation, for a distributed resel decreases as the Fourier transform of the impulse response
- For a sinc^2 impulse response, correlation falls off linearly from unity at no separation to zero at a “critical baseline”

$$B_c = \frac{\lambda R}{2\delta \cos\theta}$$

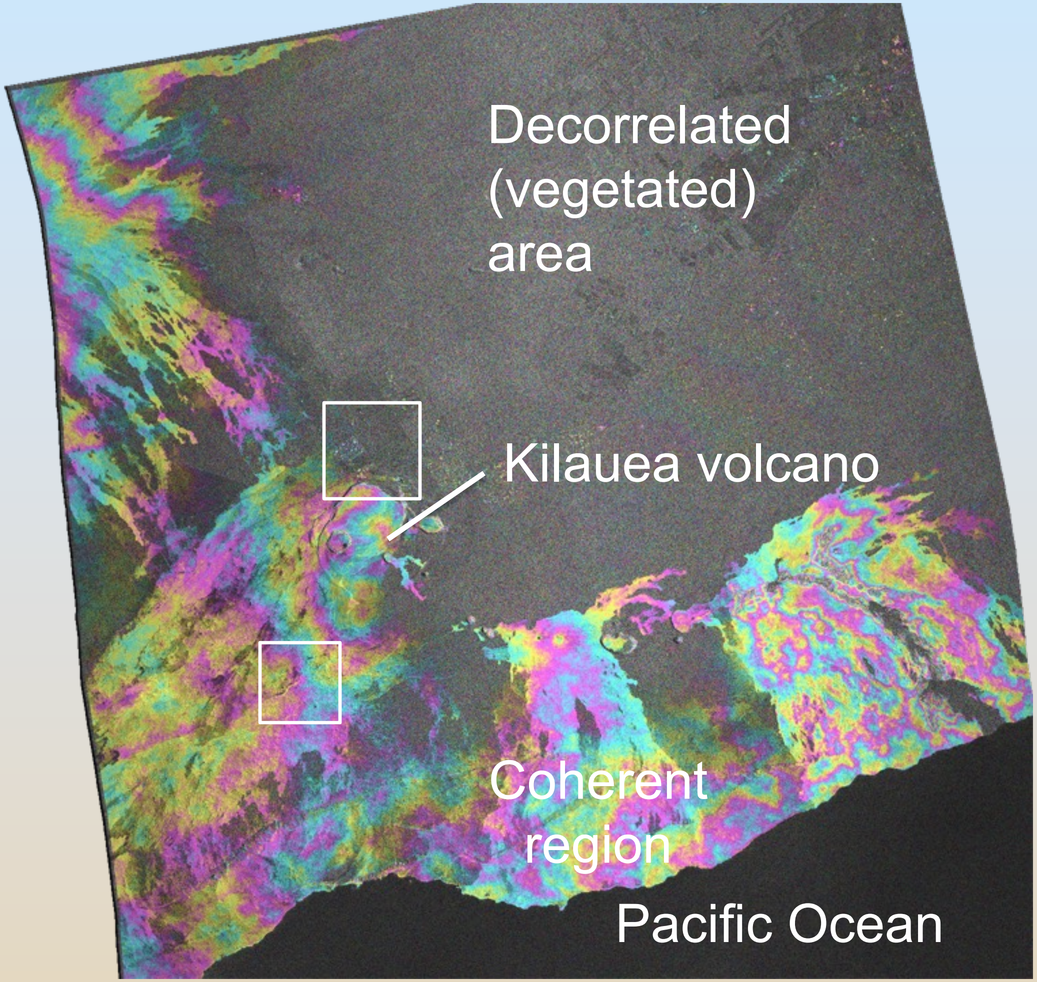
Temporal coherence

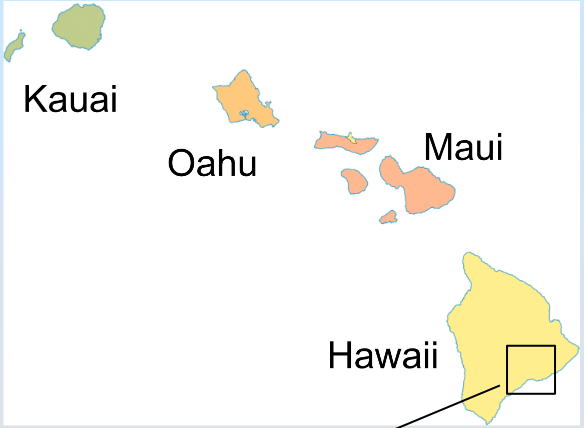
- Correlation falls off with time separation as ground surface changes with time
- Typical falloff is linear or exponential with time depending on the surface



- Decrease is faster for shorter wavelengths

X-band interferogram

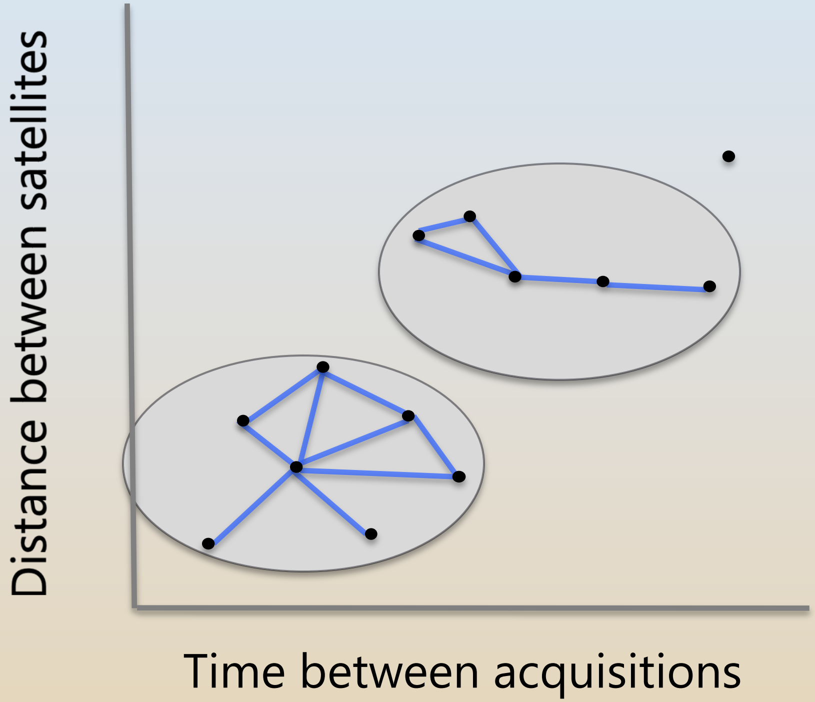




Typical interferogram of Hawaii, showing loss of coherence in heavily vegetated area

High coherence interferograms:
Small Baseline Analysis (SBAS) condition

- Plot time and baseline relation
- Choose pairs that minimize temporal and spatial decorrelation by choosing short distances



SBAS: Small earthquake displaces surface 2 cm

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