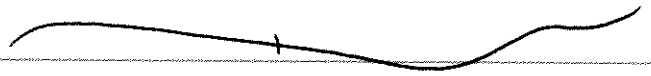
SBAS time series

$d(t)$



times

ave. velocities v_{12} v_{23} v_{34}

sequence of obs. d_i at times t_i

$$v_{ij} = \frac{d_j - d_i}{t_j - t_i}$$

Suppose we have a set of interferograms ϕ_{ij}

$$\phi_{ij} = \frac{-4\pi}{\lambda}(d_j - d_i)$$

Define a matrix $A(n_{\text{interferograms}}, n_{\text{observations}} - 1)$

For each interferogram k, create a row in A such that

~~$A(k, i : j-1) = t$~~

$$A(k, m) = t_{m+1} - t_m \quad , \quad m = i, j-1$$

Example: 3 observations at t_1, t_2, t_3

3 interferograms possible

$$A = \begin{pmatrix} t_2 - t_1 & 0 \\ 0 & t_3 - t_2 \\ t_2 - t_1 & t_3 - t_2 \end{pmatrix}$$

Then we have matrix equation

$$A \cdot v = \underline{\Phi}$$

Same example

$$\frac{-4\pi}{\lambda} \begin{pmatrix} \epsilon_2 - \epsilon_1 & 0 \\ 0 & \epsilon_3 - \epsilon_2 \\ \epsilon_2 - \epsilon_1 & \epsilon_3 - \epsilon_2 \end{pmatrix} \begin{pmatrix} V_{12} \\ V_{23} \end{pmatrix} = \begin{pmatrix} \phi_{12} \\ \phi_{23} \\ \phi_{13} \end{pmatrix}$$

$$\frac{-4\pi}{\lambda} (\epsilon_2 - \epsilon_1) V_{12} = \phi_{12}$$

$$-\frac{4\pi}{\lambda} (\epsilon_3 - \epsilon_2) V_{23} = \phi_{23}$$

$$-\frac{4\pi}{\lambda} \left[(\epsilon_2 - \epsilon_1) V_{12} + (\epsilon_3 - \epsilon_2) V_{23} \right] = \phi_{13}$$

Solve $A v = \underline{\Phi}$ by your preferred method:

$$A^{-1} A v = A^{-1} \underline{\Phi}$$

$$v = A^{-1} \underline{\Phi} \quad \text{where } A^{-1} \text{ is } \mathrm{pinv}(A)$$

or

$$v = (A^T A)^{-1} A^T \underline{\Phi} \quad \text{unconstrained least squares}$$

or $$v = (A^T A + \mu H^T H)^{-1} A^T \underline{\Phi} \quad \text{constrained least squares}$$

Least squares reminder:

Forward problem $A v = \Phi$

Measurement: $(Av - \Phi) = \epsilon$

$$(Av - \Phi)^T (Av - \Phi) = \epsilon^T \epsilon$$

$$(v^T A^T - \Phi^T)(Av - \Phi) = \epsilon^T \epsilon$$

$$v^T A^T A v - v^T A^T \Phi - \Phi^T A v + \Phi^T \Phi = \epsilon^T \epsilon$$

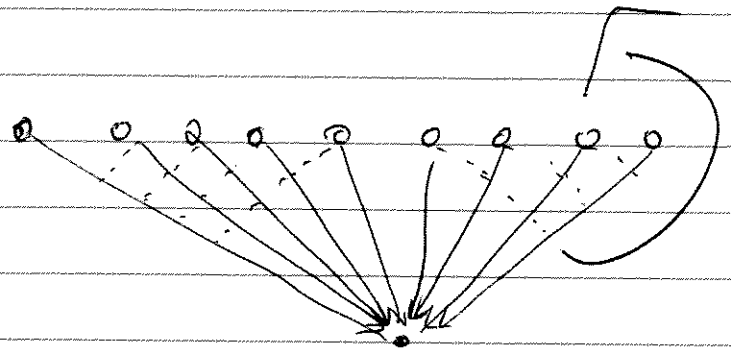
Find v to minimize error $|\epsilon|^2$, differentiate wrt V and set equal to zero:

$$2 A^T A v - 2 A^T \Phi = 0$$

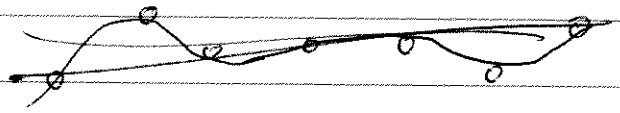
$$A^T A v = A^T \Phi$$

$$v = (A^T A)^{-1} A^T \Phi$$

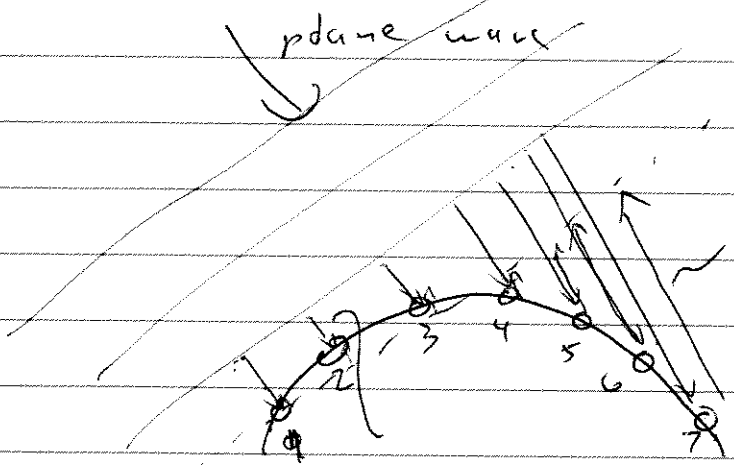
Beamforming viewpoint



→ synthetic aperture



Conformal antennas



delay = ~~ϕ~~ × cm

$= -\frac{4\pi}{\lambda} \cdot x$ rad.