EE355/GP265

Longer time scale motions

We have just shown that we can operate an interferometer that is sensitive to motions on the surface of the ocean by acquiring two images separated in time by a tenth of a second or so. If we can measure our phase to $\frac{1}{100}$ cycle or so, this means that we can measure currents at a precision of

$$\sigma_v = \frac{\lambda v}{4\pi B}\sigma_\phi \quad , \sigma_\phi \text{ in radians}$$

$$= \frac{\lambda v}{4\pi B} \cdot \frac{2\pi}{100}$$

$$= \frac{0.24 \cdot 250}{2 \cdot 20 \cdot 100} = 1.5 \text{ cm/s} \quad \text{for } \lambda = 24\text{ cm}, B = 20\text{ m}, v = 250\text{ m/s}.$$

What if instead of acquiring our two images on the same platform and pass, we use a satellite in an orbit that repeats exactly every month or so? Our velocity sensitivity could be

$$\sigma_v = \frac{\lambda}{4\pi} \frac{\sigma_\phi}{\Delta t}$$

where we have replaced $\frac{B}{v}$ by the time interval Δt. For a 24 cm system with 1/100 cycle phase accuracy, and $\Delta t = 30$ days, we have

$$\sigma_v = \frac{0.24}{4\pi} \cdot \frac{1}{30\text{days}} \cdot \frac{2\pi}{100}$$

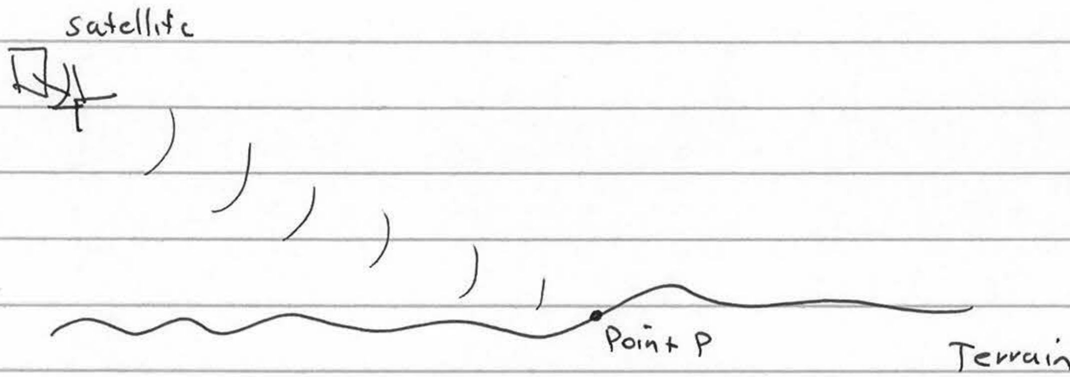
$$= 4 \times 10^{-5} \text{ m/day} \quad , \text{or} \quad 1.5 \text{ cm/year !}$$

Now, tectonic motions of the major plates on the surface of the Earth are about a few cm/yr, so we potentially can map even these very slow processes.

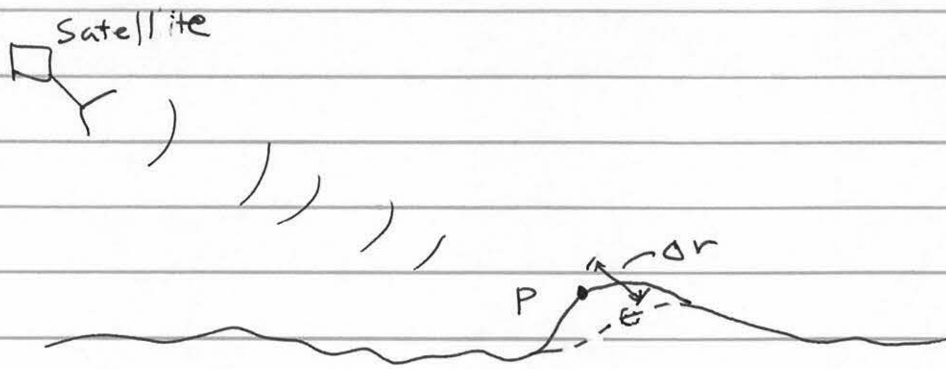
Interferometry for surface deformation

There is a more intuitive way to picture surface deformation measurements, looking at the displacements directly rather than inferring velocity. This is also important because many motions are episodic and don't occur as smooth, continuous motions.

Instead, let's consider the following experiment. Illuminate a scene at some time t_0, and record the radar image.



Now, some time later, repeat the same experiment, but displace the point P toward the radar by a distance Δr:



Something has deformed the surface in the vicinity of P and caused it to be closer to the radar at new time $t_1 = t_0 + \Delta t$. The phase of the pixel in the two scenes is

$$\phi(t_1) = \frac{-4\pi}{\lambda} r(t_1) + \phi_{scatt}$$

$$\phi(t_2) = -\frac{4\pi}{\lambda} r(t_2) + \phi_{scatt}$$

where ϕ_{scatt} is the phase associated with the random sum of the waves from each scattering center in the resolution cell. If the surface is unchanged between observations, ϕ_{scatt} will be the same in the two images. If in addition the radar is at exactly the same place in the sky at time t_2, the interferogram phase will be

$$\phi_{int} = \phi_{t_1} - \phi_{t_2}$$

$$= -\frac{4\pi}{\lambda}(r(t_1) - r(t_2))$$

$$= -\frac{4\pi}{\lambda} \Delta r$$

Thus the phase of this interferogram at every point is the line of sight component Δr of any surface motion that occurred between times t_1 and t_2.

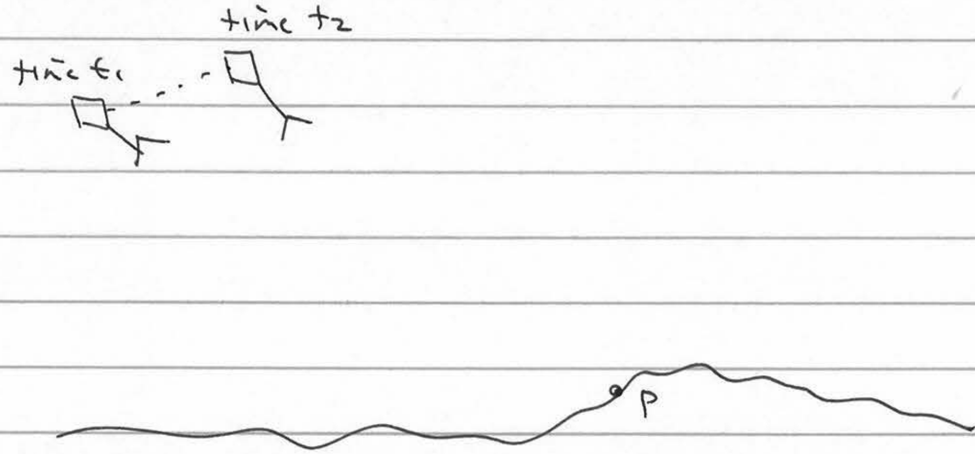
Complicating factors

Two things can confuse the above simple picture: surface changes can cause ϕ_{scatt} to be unequal in the two images, or the satellite can return to a point not quite exactly where it was on the previous pass.

Surface changes altering ϕ_{scatt} are the temporal decorrelation phenomenon we have already discussed. We can minimize the temporal effect by using shorter temporal bandwidths or longer imaging wavelengths. But both of these affect our motion sensitivity.

→ Why is that?

The second problem is associated with imprecision in the satellite orbits. Note that if the satellite returns to a spot slightly different in space to the position of the first acquisition, we reproduce our topography interferometer.



We know that the interferogram in this situation contains terms dependent on local topography. Hence even if there was motion around P, the phase signature of the deformation is mixed with that of the topography. Thus we need a method to distinguish the two types of phase modulation.

<u>Equations for deformation interferometry</u>

Begin with our equations for topography as before:

$$\phi_{int} = \frac{-4\pi}{\lambda}\left(r_1(t_1) - r_2(t_2)\right)$$

where we have explicitly identified the r's as derived from separate orbits. Thus

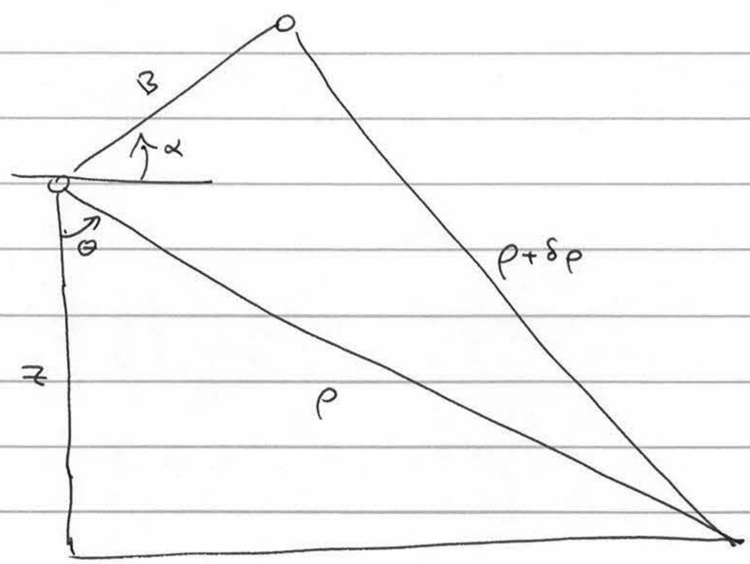
$$\phi_{int} = \frac{-4\pi}{\lambda}\,\delta\rho$$

where $\delta\rho$ is the change in range due to a difference in orbit geometry, not motion.

We used a law of cosines expansion to obtain

$$(\rho + \delta\rho)^2 = \rho^2 + B^2 - 2\rho B \sin(\theta - \alpha)$$

from the construction



Neglecting the term of order $(\delta\rho)^2$, we found

$$\delta_\rho = B\sin(\theta - \alpha) + \frac{B^2}{2\rho}$$

Furthermore, the parallel ray approximation leads to a further simplification

$$\delta\rho = B\sin(\theta - \alpha)$$

or

$$\delta\rho = B_{\parallel}$$

or that the topographic phase is proportional to the parallel component of the baseline at each point:

$$\phi_{topo} = \frac{-4\pi}{\lambda} B_{\parallel}$$

Now, suppose that in addition we have a displacement of a point P by Δr toward the radar. The interferogram phase will be altered by the corresponding amount

$$\phi_{int} = \frac{-4\pi}{\lambda} B_{\parallel} - \frac{4\pi}{\lambda} \Delta r$$

We need to be able to separate the two effects. In the initial topography case, we assumed $\Delta r = 0$, so only the first term contributed. In the velocity case, we assumed $B_{\parallel} = 0$, so only the second term mattered. How can ~~that~~ we accomodate both?

<u>Elimination of topographic phase</u>

Consider the more complete form of the phase equation where dependence on topography is shown explicitly.

$$\phi_{int} = -\frac{4\pi}{\lambda} B \sin(\theta - \alpha) - \frac{4\pi}{\lambda} \Delta r$$

and $\quad z = r \cos\theta$

$$\phi_{int} = -\frac{4\pi}{\lambda} B \sin\left(\cos^{-1}\frac{z}{r} - \alpha\right) - \frac{4\pi}{\lambda} \Delta r$$

Suppose first of all that we have independent knowledge of the topography of an area. After all, many areas of the world have been mapped. We could calculate the topography term from knowledge of z and the imaging geometry, r, B, and α. Then we could subtract this phase from the measured interferogram and we would be left with

$$\phi_{def} = \frac{-4\pi}{\lambda} \Delta r$$

This is the approach pioneered by the French at CNES and it often works well (we'll address limitations below). So if we have a map of an area in digital form with sufficient accuracy this approach is viable.

<u>Three-pass method</u>

Sometimes we do not have maps of a region available to remove the topographic term directly. In this case we can eliminate the topo dependency by using another radar pass over the scene. In essence, we use the radar to map topography and then subtract off that term.

In fact we will not have to solve for the topography explicitly to use this approach.

We'll start by altering our geometric construction to show two interferograms formed by three passes, where one pass serves as a common reference to the other two.

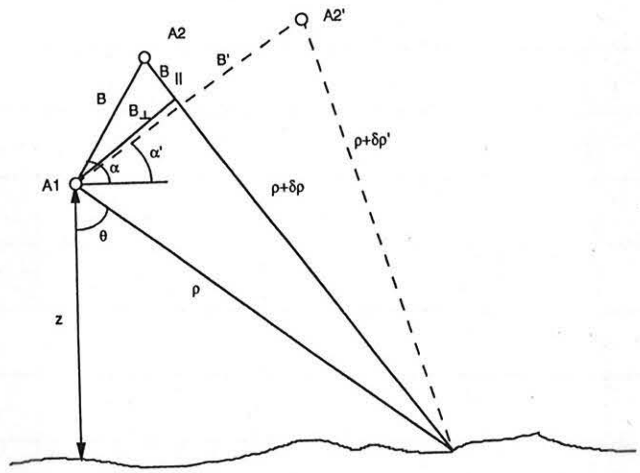


Figure 1. Radar imaging geometry. The solid lines show that radar signal paths for the first interferogram pair formed by antennas at A1 and A2. Dashed lines show signal path for second interferogram acquired over the same site but with antennas located at A1 and A2′.

The phase from the solid line system, assuming for the moment absence of deformation, was given above

$$\phi = \frac{-4\pi}{\lambda} B_{\parallel}$$

so from the primed system, with the dashed lines,

$$\phi' = \frac{-4\pi}{\lambda} B'_{\parallel}$$

In general $\phi \neq \phi'$ as the baselines are different. But, the underlying topography is constant in both pairs. So we can express the ratio

$$\frac{\phi}{\phi'} = \frac{B_{\parallel}}{B'_{\parallel}}$$

which depends only very weakly on topography.

Suppose for the moment that $\frac{\phi}{\phi'}$ or $\frac{B_{11}}{B'_{11}}$ is truly a constant value with respect to topography. Now consider how ϕ' would change if we had a surface deformation occurring ~~at~~ during the interval we collect the primed interferogram over but no displacement over the unprimed interval. Then

$$\phi' = \frac{-4\pi}{\lambda}\left(B'_{11} + \Delta r\right)$$

and since $\phi = \frac{-4\pi}{\lambda} B_{11}$, if we form the quantity

$$\phi' - \frac{B'_{11}}{B_{11}}\phi = \frac{-4\pi}{\lambda}\Delta r$$

Thus if we scale the unprimed measurement by the ratio of the parallel baselines, and subtract it from the primed interferogram, we obtain the deformation interferogram directly.

Thus if we use three radar passes, we get the same result as if we had used the two pass plus map method.