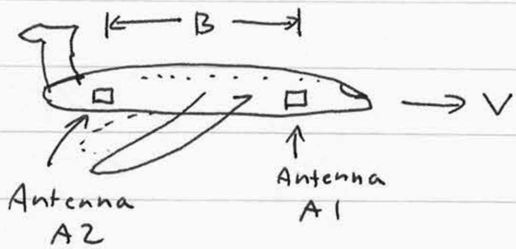
## Sensing motions

Today we will change our interferometer geometry to enable us to detect motions rather than topography. We will use many of the same processing techniques described in previous lectures but the imaging geometry will ensure that our baseline is described in time, rather than space.

## Temporal baselines

To accommodate temporal measurements, our interferometer baseline must span time rather than space. Consider the following experimental setup:




The radar antennas A1 and A2 are mounted on the front and back sections of an aircraft fuselage, rather than across-track. We will form two radar images again, as we did in the earlier interferometer.

Suppose the aircraft flies at velocity $v$, and than the antennas are displaced by a distance $B$ along track. Other than thermal noise effects, the two images formed in the two channels will be identical, except that the channel two image is delayed in time by the time it takes the aircraft to fly a distance $B$, $\frac{B}{V}$.

Expressing the image coordinates along-track and across track by $x$ and $y$, as usual,

$$i_1(x,y) = i_2(x+B, y)$$

In azimuth time and slant range,

$$i_1(t,r) = i_2\left(t+\frac{B}{v}, r\right)$$

In both cases the images are identical except for a shift in position along-track. Now, suppose we are observing an object whose range from the flight track, not the sensor, is $r(t)$. For a stationary object $r(t)$ is constant.

Once again form an interferogram by multiplying $i_1$ by $i_2^*$, but shift the images to overlap so as to align scatterers between the images. We'll need to delay image $i_2$ by $\frac{B}{v}$, so

$$i(t,r) = \cancel{i_1(t,r) * i_2(}$$

$$i_1(t_1, r(t_1)) \cdot i_2^*(t_2, r(t_2))$$

$$= i_1(t_1, r(t_1)) \cdot i_2^*\left(t_1+\frac{B}{v}, r\left(t_1+\frac{B}{v}\right)\right)$$

Note that the phase of the object at $t$ is

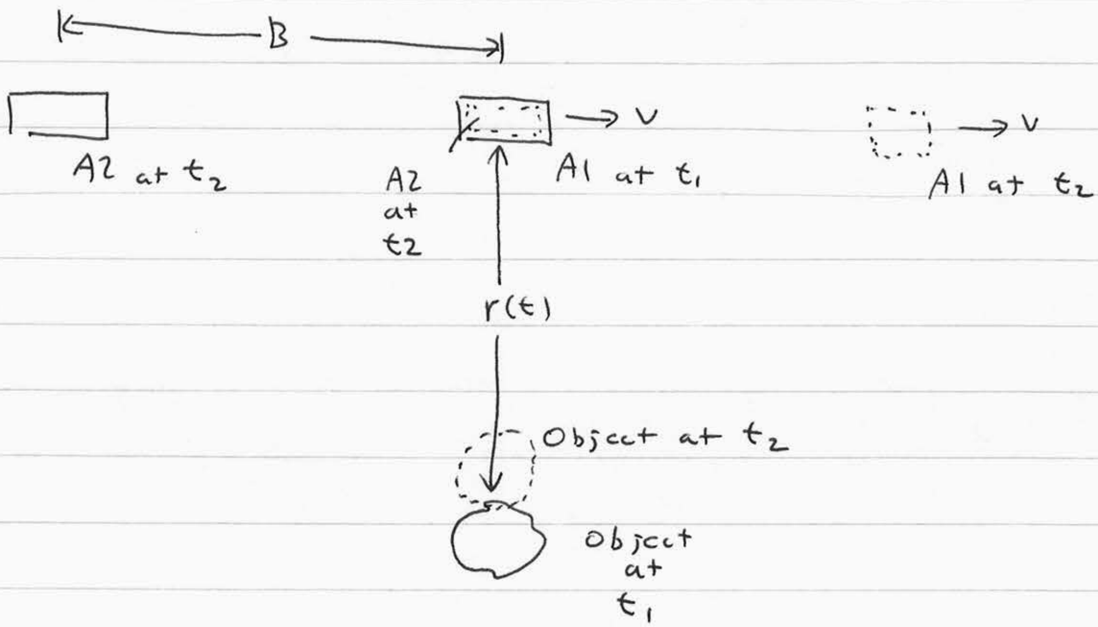
$$\phi = \frac{-4\pi}{\lambda} r(t)$$

hence the interferogram phase $\phi_1 - \phi_2$ is

$$\phi_1 - \phi_2 = \frac{-4\pi}{\lambda}\left(r(t_1) - r\left(t_1+\frac{B}{v}\right)\right)$$

If the object is not moving, $\phi_1 - \phi_2 = 0$, or the image is identical at both times.

Now what happens if the object is moving?




Suppose the object is moving toward the radar at velocity $\upsilon$. Then

$$r(t+\Delta t) = r(t) - \upsilon \Delta t$$

and the interferogram phase $\phi_{int}$ is

$$\phi_{int} = \frac{-4\pi}{\lambda}\left[ r(t) - r\left(t + \frac{B}{V}\right)\right]$$

$$= -\frac{4\pi}{\lambda}\left[ r(t) - \left\{ r(t) - \upsilon \frac{B}{V} \right\}\right]$$

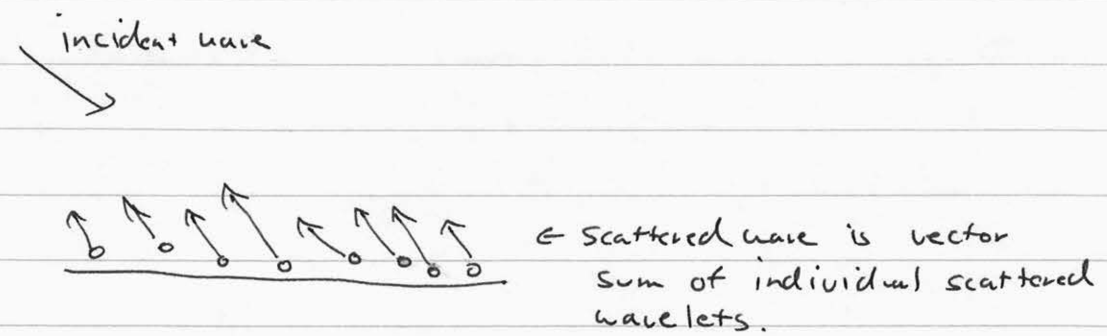
$$= -\frac{4\pi}{\lambda} \cdot \frac{\upsilon}{V} \cdot B$$

Thus the phase is directly proportional to the velocity of the object $\upsilon$.

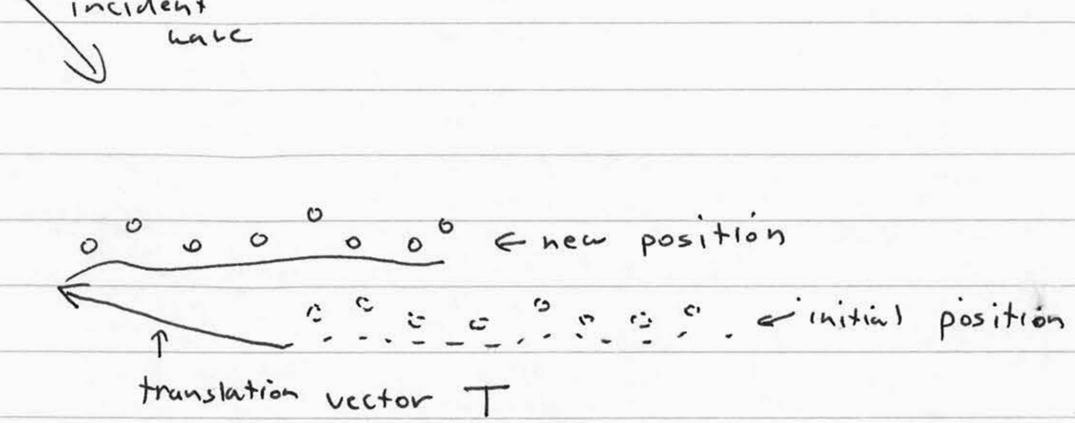
## Ocean Currents

We have previously gone through a detailed argument showing that motion of the surface leads to decorrelation. Won't this target movement also lead to decorrelation?

Consider a set of scatterers as we had before for the discrete scattering model:




If the incident direction changes significantly (by the critical baseline) or if we allow random motion among the scatterers, we will observe decorrelation. But if we simply translate the full set of scatterers the distance to each scatterer changes by the same amount.




Thus the mean value of the phase of the echo changes, but the higher order moments are unchanged, and we get no decorrelation.

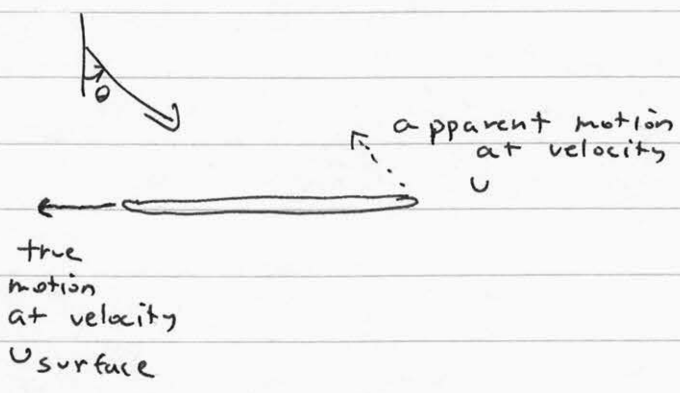
It should also be clear that the amount of phase shift is proportional to the component of the translation vector T that lies in the radar line of sight direction. Hence only one component of T is measured.

Therefore the value of $u$ that is inferred from

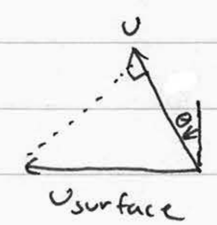
$$\phi = \frac{4\pi B}{\lambda} \frac{u}{V}$$

is the component in the look direction.

What does this mean for ocean current and wave imaging? For currents the entire surface moves together, so the interpretation is straight-forward. For the ocean a good approximation is that the surface is flat, so that motions are constrained to lie in a plane. This makes for easy interpretation of our measured $u$ in terms of true surface velocity.




so the following construction works




$$\frac{u}{u_{surface}} = \sin\theta$$

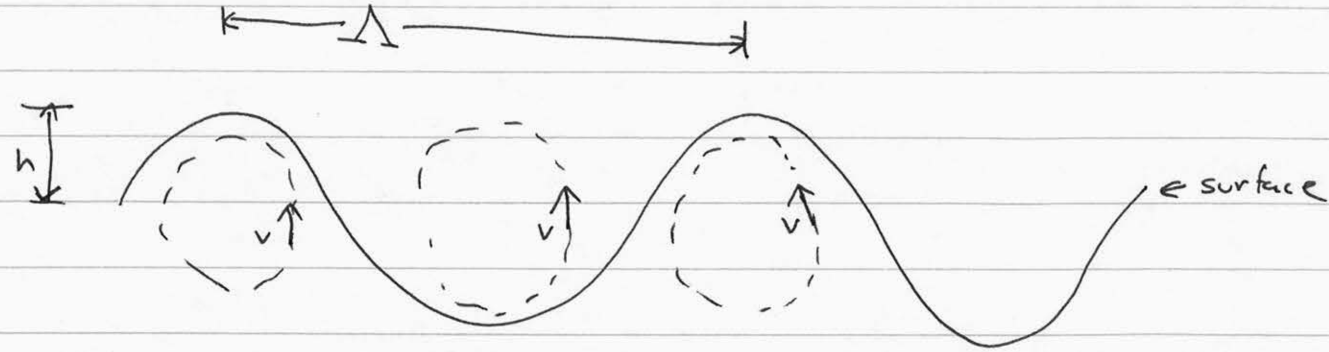
$$u_{surface} = \frac{u}{\sin\theta}$$

Thus our solution for surface velocities for ocean currents is

$$U_{surface} = \frac{\phi \lambda V}{4\pi B \sin\theta}$$

Swell - Large-scale ocean waves

A somewhat different situation applies if we image fields of large (compared to a resel) ocean waves, called swell. In that case individual parcels of water move in circular, or orbital, motions, as in:




For large-scale ocean waves, called gravity waves because gravity forms the restoring force, we can relate orbital motion to wavelength by considering basic wave equations:

$$c = \sqrt{\frac{\Lambda g}{2\pi}} \quad \leftarrow \text{phase velocity of wave}$$

and the period

$$T = \frac{\Lambda}{c}$$

Now, the orbital motion is such that a parcel of water traverses a circle of radius h in the same period, so the orbital velocity $V_{orb}$ is

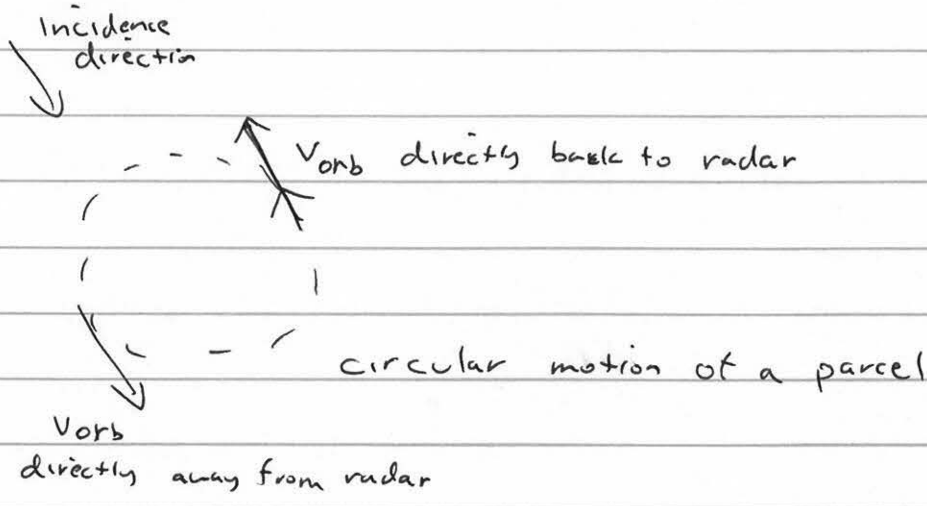
$$V_{orb} = \frac{2\pi h}{T}$$

Combining these, we get

$$V_{orb} = \frac{2\pi h c}{\Lambda} = \frac{2\pi h}{\Lambda}\sqrt{\frac{\Lambda g}{2\pi}}$$

$$= \sqrt{\frac{2\pi g h^2}{\Lambda}}$$

Note that we don't have the same obliquity problem with swell velocity that we had with surface currents, as the motions are circular and not constrained to lie in a plane.

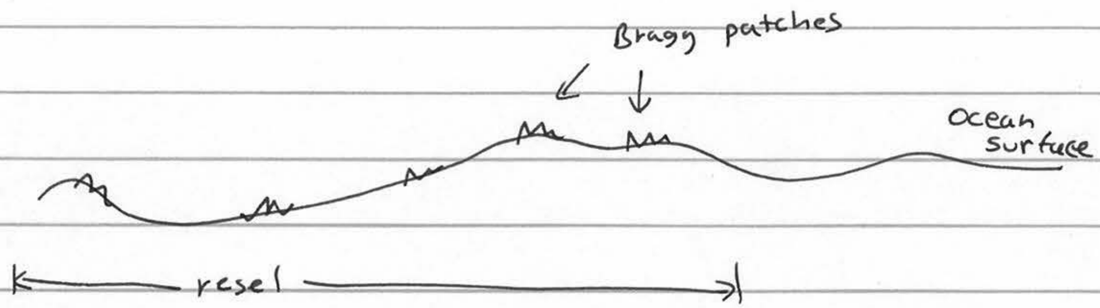



Now, since we can find $V_{orb}$ directly from the interferogram phase, and $\Lambda$ from the intensity image or phase, we can solve for $h$, the height of the swell. Hence using an interferometer for ocean remote sensing characterizes swell nicely.

## Ocean scattering models

We have implicitly assumed that our discrete scatterer model was appropriate for the ocean surface in order to derive our interferometric relations. But it may be hard to picture the physical realization of these scattering centers on the surface of water.

A useful model for ocean surface scattering for interferometric purposes is to use the discrete scatterer approach, but form the scatterers by created small "patches" which scatter by the Bragg mechanism. These Bragg patches of matched radar wavelength are then scattered about the resolution element to complete the scattering picture.



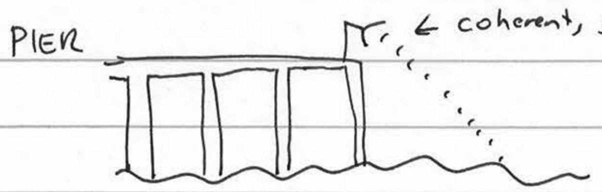

Each patch consists of a few cycles of wavlets with the Bragg condition

$$\Lambda_{Bragg} = \frac{\lambda}{2\sin\theta}$$

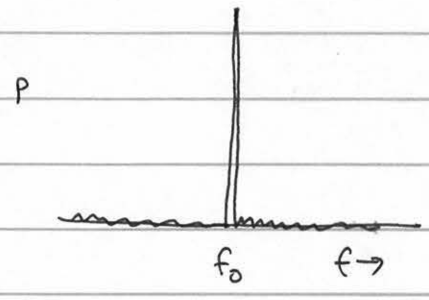
Two issues are raised by incorporating this model into interferometry. One, we have to worry about the lifetime of the scattering patches. Clearly a given patch on a real water surface will dissipate with time. Our temporal interferometer baseline cannot exceed this lifetime.

Second, because these patches are waves they too will be moving and will bias our velocity results. We certainly must correct for this effect in interpreting our velocity field.

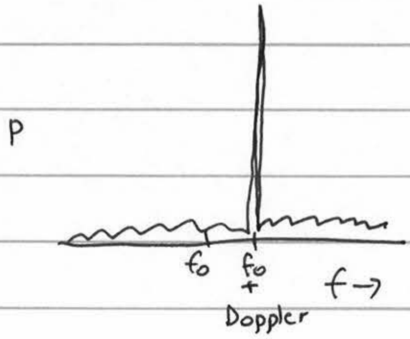
Lifetime - Several groups have measured the lifetime of Bragg patches by using single-frequency coherent measurement techniques. The experiment is as follows:




Consider the spectrum of the received echo. If we were looking at an unmoving land surface, the received spectrum would be very narrow, like this



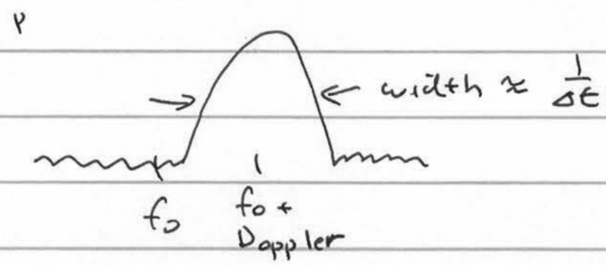

Now suppose we are looking at a series of swell or a fixed current. The entire spectrum will be Dopper shifted to reflect the motion toward the radar.



The width of the spectrum will be determined by the integration time in our receiver, and depicts the usual time-frequency uncertainty relation

$$\Delta t = \frac{1}{\Delta f}$$

What happens if the patch lifetime is less than our integration time? In this case the observed width of the spectrum will indicate the lifetime rather than the observation time. In other words, the integration time is limited by the coherence time, or lifetime, of the patch. Thus the spectrum will be




Such measurements seem to indicate lifetimes of ~ ½ sec for L band, ~ 0.1 sec for C-band for nominally smooth ocean states. It would decrease for rougher seas.

Is this long enough for us? A typical baseline might be 20 m for a jet aircraft, hence

$$\frac{B}{V} = \frac{20\ m}{250\ m/s} = 0.08\,s$$

which would be comfortably within the L-band limit but pushing things for C-band. A C-band instrument should have a smaller baseline, say two meters or so.

<u>Motion effects</u> Because the Bragg patches are composed of waves, they will be in motion on the water, either toward or away from the radar. Using

$$c = \cancel{\sqrt{\frac{\Lambda g}{2\pi}}} = \sqrt{\frac{\Lambda g}{2\pi}}$$

and the Bragg wavelength $\Lambda = \frac{\lambda}{2 \sin\theta}$

$$c_{Bragg} = \sqrt{\frac{\lambda g}{4\pi \sin\theta}}$$

so for L band ($\lambda = 24$ cm), and $30°$ incidence,

$$c_{Bragg} = 0.6 \text{ m/s}$$

so the patches themselves will have velocities on the surface of $\pm 0.6$ m/s. The line-of-sight component will be either 0.3 m/s or -0.3 m/s. This motion will be seen as a bias on top of the desired current or swell velocity.

Hence, we measure the algebraic sum of all three of these effects, and each must be considered in any oceanographic application. But the ability to make these measurements at the resel scale for large swaths makes the instrument appropriate for a great many ocean studies.