Decorrelation

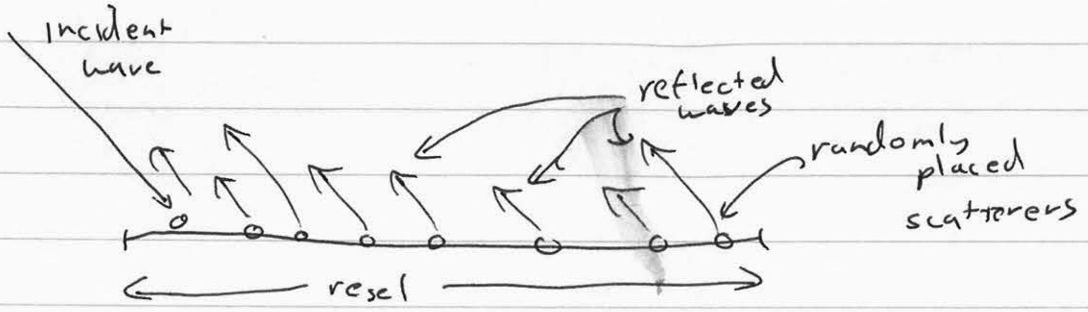
We mentioned, almost in passing, the correlation of a radar interferogram last time. In fact correlation is quite fundamental to understanding and interpreting interferometic measurements.

Correlation is closely coupled with our ability to estimate interferometric phase accurately, and hence use the data in practical applications. So, we will now take a closer look at it and its various sources in a radar system.

We will consider radar echoes to be "correlated" with each other if the measured phase and amplitudes coincide and hence represent the "same" interaction of radar signals and scattering terrain. In an imaging radar, this means that the observed "speckle" patterns are nearly the same. So first we need to understand speckle in a radar image.

Speckle

Speckle is the observable "graininess" in a radar image that is due to EM interactions rather than actual variations in σ° of a surface. Consider the echo from a single resel where we use the discrete scattering model from a handful of scatterers:



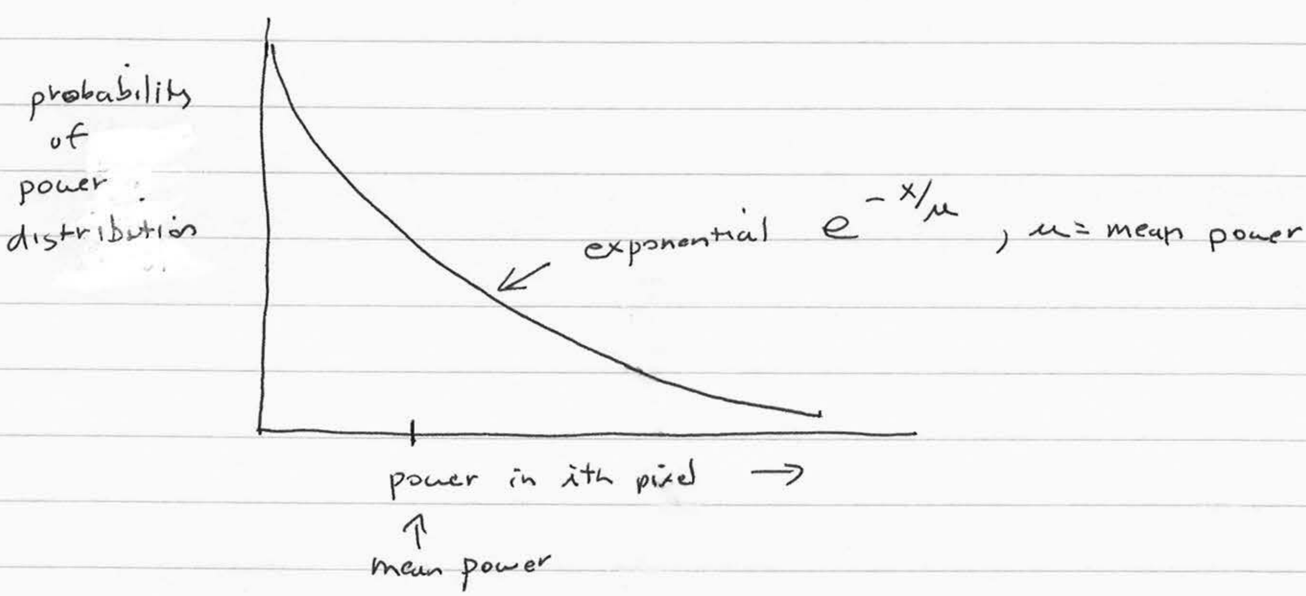
The received echo is the coherent sum of the reflected waves:

$$r(t) = \sum_{\substack{n \\ \text{scatterers}}} a_i e^{-j \frac{4\pi}{\lambda} r_i}$$

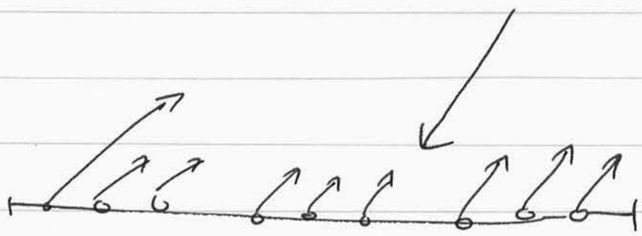
where a_i is the amplitude of the ith scatterer and r_i is the range to the ith scatterer.

If all the r_i are equal, the signals add in phase (coherently) for a large echo. In our previous figure, assuming a flat surface, that would be the case and we'd get a strong specular return.

But, since the resel size is typically much greater than a wavelength, the phase associated with r_i is essentially uniformly distributed over the interval $(0, 2\pi)$. Hence the waves will all interfere with each other, resulting in some pixels with a great degree of cancellation (near zero return) and some pixels where a lot of coherent addition occurs (bright return). The average power is well-related to the number of scatterers and the cross-section of each, but the realization of any scattering event is Rayleigh distributed in amplitude, or exponential in power.



Now, suppose we image the same set of scatterers from an entirely different viewing angle:



All of the distances r_i are different, hence each scatterer adds with a different phase than before. Thus, even though the same set of scatterers is illuminated, the phase will be entirely different. Thus changing viewing angle by too much makes the interferometric phase all noise, and no coherent analysis is possible.

Decorrelation theory

Consider two radar signals s_1 and s_2 acquired by two interferometric antennas, very near each other but going through separate receivers. We can model the signals as consisting of a common signal component c and individual noises n_1 and n_2:

$$s_1 = c + n_1$$
$$s_2 = c + n_2$$

Then the correlation by our previous formula is

$$\rho_{thermal} = \frac{\langle s_1 s_2^* \rangle}{\sqrt{\langle s_1 s_1^* \rangle \langle s_2 s_2^* \rangle}}$$

where $\langle \rangle$ denotes ensemble averaging and the subscript "thermal" shows the noise is due to thermal properties.

Because c, n_1, and n_2 are uncorrelated random variables,

$$\rho_{thermal} = \frac{|c|^2}{|c|^2 + |n|^2}$$

Using $SNR = \frac{|c|^2}{|n|^2}$,

$$\rho_{thermal} = \frac{1}{1 + \frac{1}{SNR}}$$

which gives the relation of correlation to SNR.

Next, we introduce some additional decorrelation by changing the viewing angle slightly. As we discussed above, the individual echoes traveling distances r_i will add slightly differently, yielding a slightly different sum. We'll assume for now that the viewing angle changed only slightly, and we can model the two signals in this case as

$$s_1 = c + d_1 + n_1$$

$$s_2 = c + d_2 + n_2$$

where c again is the correlated part of the signal, n_i are the thermal noise components, and d_i represent additive "noise" due to the change in viewing direction. For reasons apparent later, we will call d_i the ~~spatial~~ <u>spatial decorrelation</u> terms, since they involve spatial movement of the antennas.

Using the same argument as above, consider the effective SNR from spatial decorrelation with no thermal noise term present:

$$\rho_{spatial} = \frac{|c|^2}{|c|^2 + |d|^2}$$

where we now have the subscript "spatial" instead of "thermal."

If we also include the thermal effects:

$$\rho_{spatial + thermal} = \frac{|c|^2}{|c|^2 + |d|^2 + |n|^2}$$

since c, d_i, and n_i are uncorrelated rvs. Let's rewrite as

$$\rho_{spatial + thermal} = \frac{|c|^2}{|c|^2 + |d|^2} \cdot \frac{|c|^2 + |d|^2}{|c|^2 + |d|^2 + |n|^2}$$

and, defining SNR as the ratio of all non-thermal to thermal powers, or $SNR = \frac{|c|^2 + |d|^2}{|n|^2}$

$$\rho_{spatial + thermal} = \rho_{spatial} \cdot \rho_{thermal}$$

$$= \frac{|c|^2}{|c|^2 + |d|^2} \cdot \frac{1}{1 + \frac{1}{SNR}}$$

<u>Thus the total correlation is the product of the individual correlations.</u>

Finally, we want to consider correlation when the two surfaces are imaged at different times. Even though the backscattered phase is a random quantity because it is the sum of the many echoes, each at distance r_i, if the scene area is unchanged between observation times, and the radar is at the same location, each r_i is unchanged and the echoes will be completely correlated. If the surface has changed slightly, though, the echoes will be a little different.

How might a surface change between observations?

<DISCUSSION QUESTION>

We can model the decorrelation from slightly changed surfaces by introducing a term in the echo corresponding to surface change. A little thought and repetition of the previous argument leads to a formula of the form

$$\rho_{total} = \rho_{thermal} \cdot \rho_{spatial} \cdot \rho_{temporal}$$

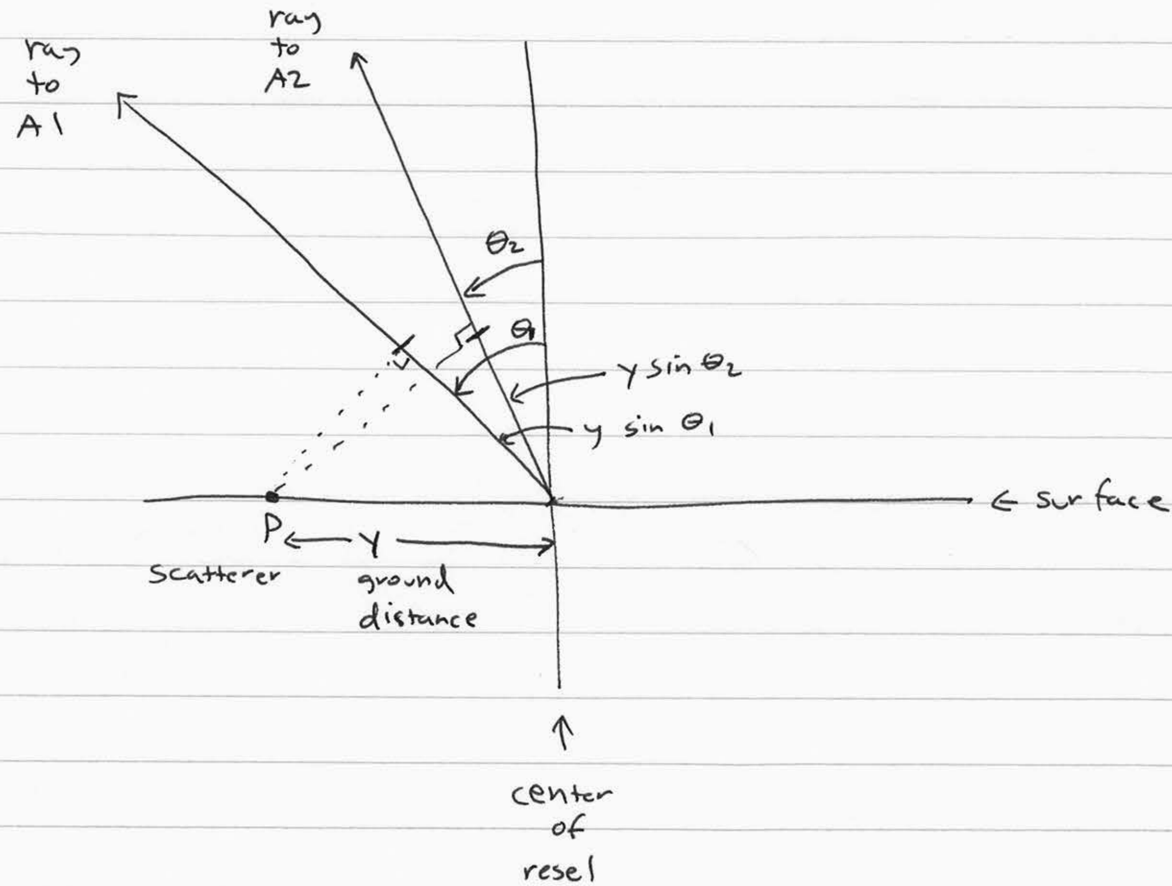
where $\rho_{temporal}$ represents decorrelation from a change in the surface between observations. $\rho_{spatial}$ referred to a change in sensor location, while $\rho_{thermal}$ modeled electronic changes inside the radar system.

Calculation of Spatial Baseline Decorrelation

Lets quantify the above discussion of $\rho_{spatial}$ and determine just how close the two interferometer antennas need to be ~~location~~ located for accurate phase estimation.

We begin by deriving a Fourier transform relation between the radar impulse response and the correlation as a function of difference in viewing angles.

Here's our imaging geometry:



Consider the phase of the reflection from P as compared to the reflection from the center of the resel. The phase from P is advanced by $y \sin\theta_i$ for the two antennas. If we model the total signal s_1 at antenna 1 as the integral over all of the resolution element, then

$$s_1 = \iint f(x-x_0, y-y_0) \exp\left(-j \frac{4\pi}{\lambda}(r + y\sin\theta_1)\right) \cdot W(x,y)\, dx\, dy + n_1$$

where $f(x,y)$ represents the complex backscatter from each point on the surface, and $W(x,y)$ is the system impulse response.

In other words, we simply add all the contributions from the points in the resel weighted by the impulse response - our usual linear system theory.

Similarly,

$$s_2 = \iint f(x-x_0, y-y_0)\exp\left(-j\frac{4\pi}{\lambda}(r+y\sin\theta_2)\right) w(x,y)\,dx\,dy + n_2$$

In both cases r is the range to the center of the pixel -- we'll let it be identical in s_1 and s_2. If this were not the actual configuration, only the mean phase of the return would change, but our conclusion below would remain unchanged.

Now, form the interfergram by computing $s_1 s_2^*$:

$$s_1 s_2^* = \iiiint f(x-x_0, y-x_0)\, f^*(x'-x_0, y'-y_0) \cdot \exp\left(-j\frac{4\pi}{\lambda} y(\sin\theta_1 - \sin\theta_2)\right)$$

$$\cdot\, w(x,y)\, w^*(x',y')\, dx\, dy\, dx'\, dy'$$

Now, if the surface scatterers are arranged randomly, uniformly distributed and uncorrelated,

$$\langle f(x,y) f^*(x',y') \rangle = \sigma^0\, \delta(x-x', y-y')$$

and we can reduce the above four-fold integral to

$$\langle s_1 s_2^* \rangle = \sigma^0 \iint e^{-j\frac{4\pi}{\lambda} y\cos\theta\, d\theta}\, |w(x,y)|^2\, dx\, dy$$

where θ is the average of θ_1 and θ_2 and $d\theta$ is their difference. Since the exponential kernel is linear in y, it can be viewed as a scaled Fourier transform relating the correlation function $\langle s_1 s_2^* \rangle$ to the power impulse response $|W(x,y)|^2$.

If the impulse response is approximated as a sinc function in both dimensions, usually a good approximation for high time bandwidth chirped systems, the transform may be evaluated and we find

$$\rho_{spatial} = 1 - \frac{2 \cos \theta \, \delta_y \, d\theta}{\lambda}$$

where δ_y is the ground range resolution $\left(\frac{\delta r}{\sin \theta} \right)$.

Since

$$d\theta = \frac{B_\perp}{r}$$

we can also use

$$\rho_{spatial} = 1 - \frac{2 \cos \theta \, B_\perp \delta_y}{\lambda r}$$

Note that this is a linearly decreasing function of $B_\perp$, going from 1 at $B_\perp = 0$ (perfect correlation) to zero at a "critical" value

$$B_c = \frac{\lambda r}{2 \cos \theta \, \delta_y}$$

If the perpendicular baseline $B_\perp$ approaches or exceeds B_c, no correlation is observed. This quantifies our observation that the interferometer antennas must be located near each other.

Rotation decorrelation

If the radar flight tracks are rotated with respect to each other between observations, a second source of decorrelation due to imaging geometry occurs, and can be quantified by

$$\rho_{rotation} = 1 - \frac{2 \sin\theta \; d\phi \; \delta_{az}}{\lambda}$$

(see Decorrelation paper for derivation)

Temporal Decorrelation

Finally, we can model the temporal decorrelation term as three-D motions of each of the individual scatterers, leading to the impressive-looking integral

$$s_1 s_2^* = \int\int\int\int\int\int f(x - x_0, y - y_0, z - z_0) f^*(x' - x_0, y' - y_0, z' - z_0)$$
$$\exp\{-j\frac{4\pi}{\lambda}(\delta y \sin\theta + \delta z \cos\theta)\} W(x, y) W^*(x', y') dx dy dz dx' dy' dz'$$

$$\langle s_1 s_2^* \rangle = \sigma^0 \int\int \exp\{-j\frac{4\pi}{\lambda}(\delta y \sin\theta + \delta z \cos\theta)\} p_y(\delta y) p_z(\delta z) d\delta y d\delta z$$

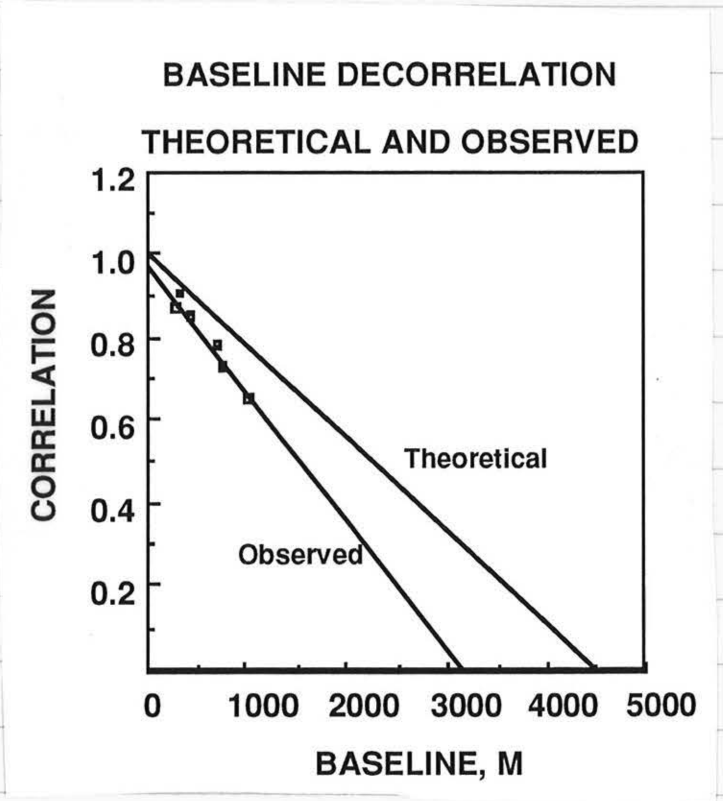
($p_y(\delta_y)$ and $p_z(\delta_z)$ are probability distributions of motions in y and z directions)

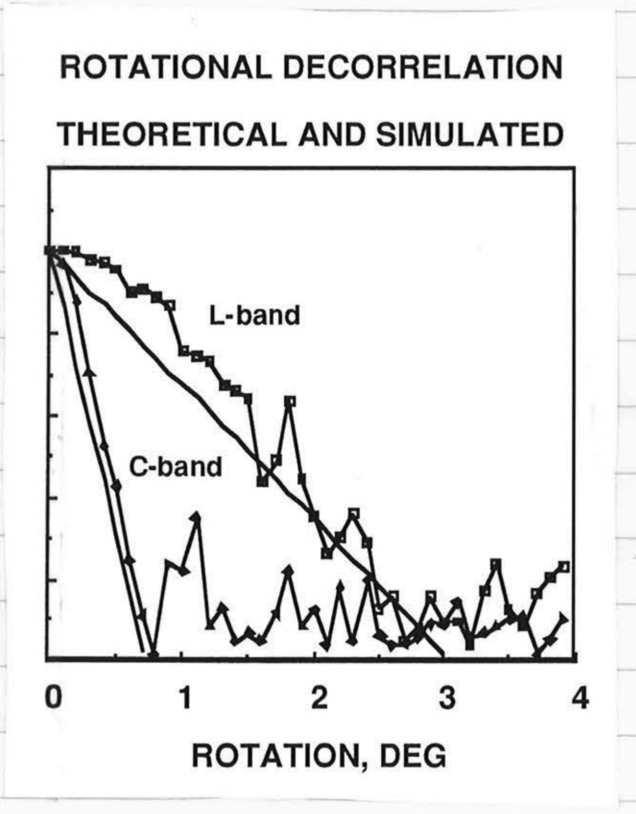
and its solution

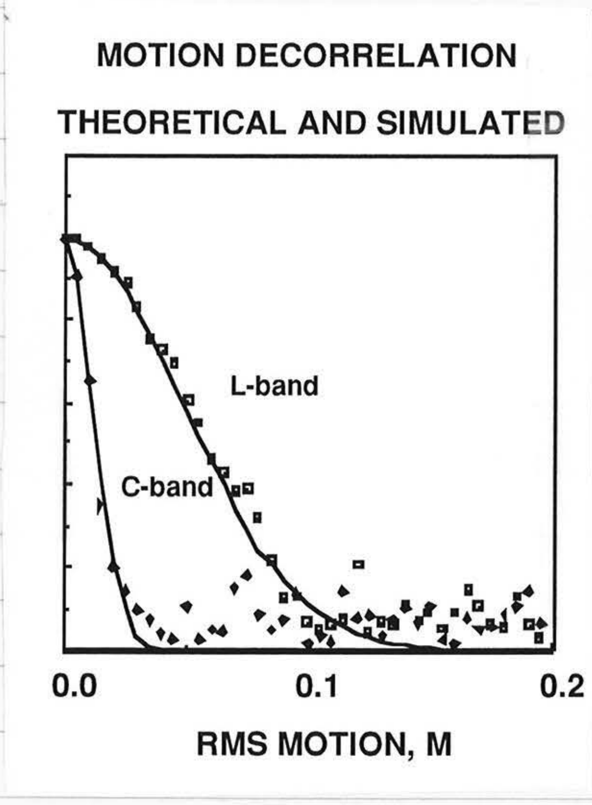
$$\rho_{temporal} = \exp\left(-\frac{1}{2}\left(\frac{4\pi}{\lambda}\right)^2 (\sigma_y^2 \sin^2\theta + \sigma_z^2 \cos^2\theta)\right)$$

where σ_y^2 and σ_z^2 are variances of motions in y and z.

Sample data and simulations of these are given below:







Finally, here is an observed determination of temporal decorrelation at L-band: