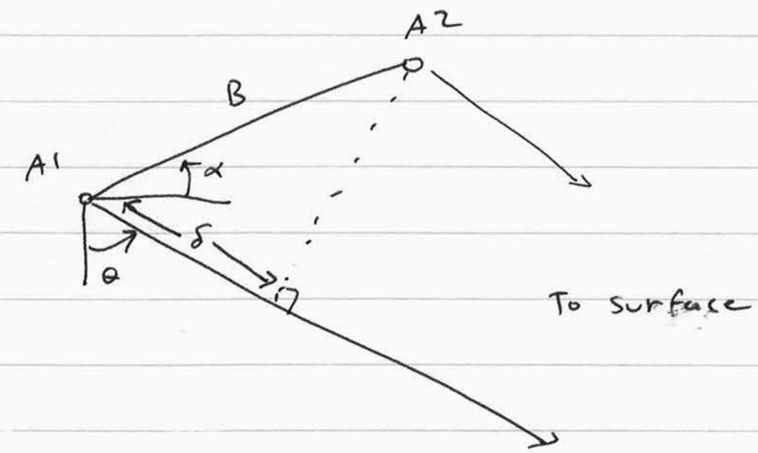
Forming the interferogram

We saw last time that if we implement an interferometer by using two antennas displaced across-track, then the phase of the difference signal from the two antennas contains information about the surface topography. Let's now examine in detail formation of such an interferogram.

From before, we had the following setup:



and we related the distance δ to the geometry by

$$\delta = B \sin(\theta - \alpha)$$

The path from A1 to the surface is δ longer than the corresponding path from A2, and δ depends on both the imaging geometry and the topography. This means that each pixel in the image from antenna A1 is displaced by 2δ in range from the corresponding pixel in the image from antenna A2. The total displacement is 2δ due to two-way propagation.

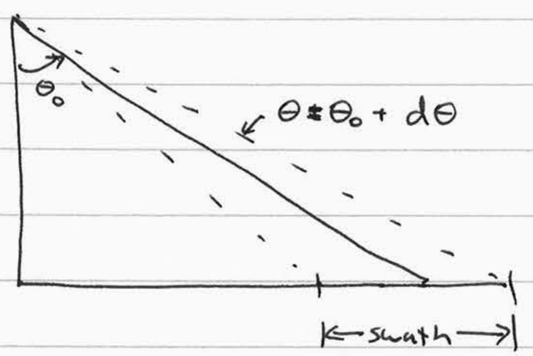
For the time being, let's neglect topography by assuming a flat surface, and investigate the form of the displacement across the swath.

Displacement vs. range

Starting with $\delta = B \sin(\theta - \alpha)$, we can relate θ to range and altitude through

$$z = r \cos \theta$$

We can solve for δ as a function of range, $\delta(r)$, by substituting the second expression into the former. But we will find it simpler to expand the result about a nominal look angle θ_0 first:



Hence

$$z = r \cos(\theta_0 + d\theta)$$

$$\approx r \left[\cos\theta_0 - \sin\theta_0 \, d\theta \right]$$

Also

$$\delta = B \sin(\theta_0 - \alpha + d\theta)$$

$$= B \left[\sin(\theta_0 - \alpha) + \cos(\theta_0 - \alpha) \, d\theta \right]$$

Eliminating $d\theta$:

$$z = r\cos\theta_0 - r\sin\theta_0\, d\theta$$

$$r\sin\theta_0\, d\theta = r\cos\theta_0 - z$$

$$d\theta = \frac{r\cos\theta_0 - z}{r\sin\theta_0}$$

and

$$\delta = B\sin(\theta_0-\alpha) + B\cos(\theta_0-\alpha)\,\frac{\cos\theta_0 - z/r}{\sin\theta_0}$$

$$= B\sin(\theta_0-\alpha) + \frac{B\cos(\theta_0-\alpha)}{\tan\theta_0} - \frac{B\cos(\theta_0-\alpha)}{\sin\theta_0}\,\frac{z}{r}$$

Now, expanding the last term about r_0 where $r = r_0 + dr$:

$$\frac{z}{r} = \frac{z}{r_0+dr}$$

$$\approx \frac{z}{r_0}\left(\frac{1}{1+\frac{dr}{r_0}}\right) \approx \frac{z}{r_0}\left(1-\frac{dr}{r_0}\right)$$

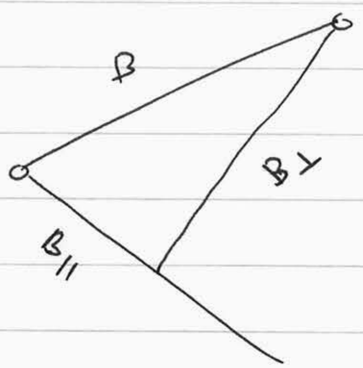
$$= \frac{z}{r_0} - \frac{z\,dr}{r_0^2}$$

So, for $\delta(dr)$ we get

$$\delta = B\sin(\theta_0-\alpha) + \frac{B\cos(\theta_0-\alpha)}{\tan\theta_0} - \frac{B\cos(\theta_0-\alpha)}{\sin\theta_0}\left(\frac{z}{r_0} - \frac{z}{r_0^2}dr\right)$$

$$= B\sin(\theta_0-\alpha) + \frac{B\cos(\theta_0-\alpha)}{\tan\theta_0}\,\frac{dr}{r_0}$$

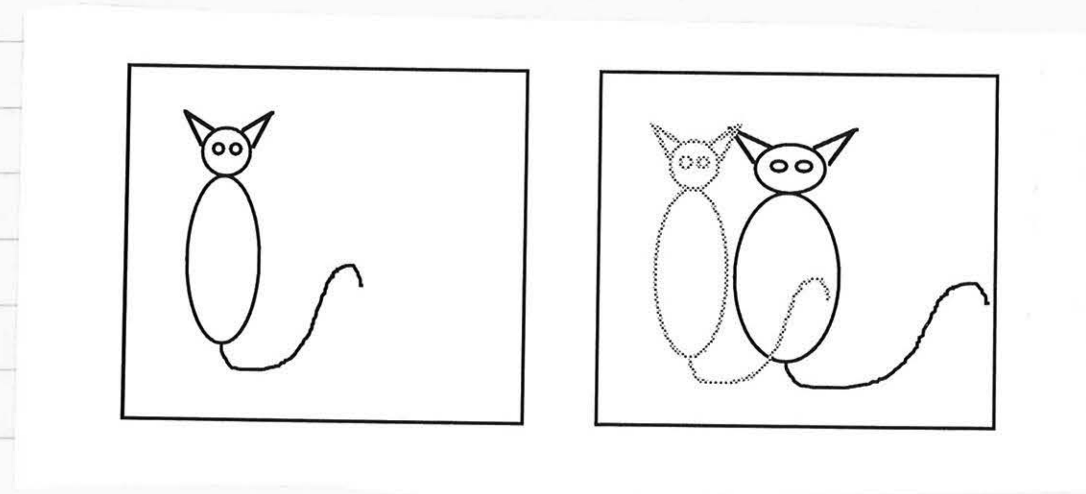
We can define the baseline at the nominal look angle θ_0 as having a parallel component $B_{\parallel} = B \sin(\theta_0 - \alpha)$ and a perpendicular compont $B_{\perp} = B \cos(\theta_0 - \alpha)$:



so $$\delta = B_{\parallel} + B_{\perp} \frac{dr}{r_0 \tan \theta_0}$$

All of this is a long way of saying that δ has a constant shift equal to the parallel component of the baseline, and a linearly increasing shift (stretch) dependent on $B_{\perp}$ and the geometrical factor $r_0 \tan \theta_0$.

A comparison of the images would then look like:



Antenna 1 image

Antenna 2 image - shifted and stretched

Now, forming the interferogram means relating the phase in a pixel in image 1 to the corresponding phase in ~~pixe~~ image 2. Thus, it is necessary to identify each pixel in image 2 in terms of its location in image 1.

We usually don't know the relationship for δ a priori, so we typically have to determine the function from our data. We do this by estimating the shift in image 2 by identifying corresponding points in the two images and solving for an equation for δ.

This is called <u>determining the interferometric offset field</u>.

<u>Offset determination</u>

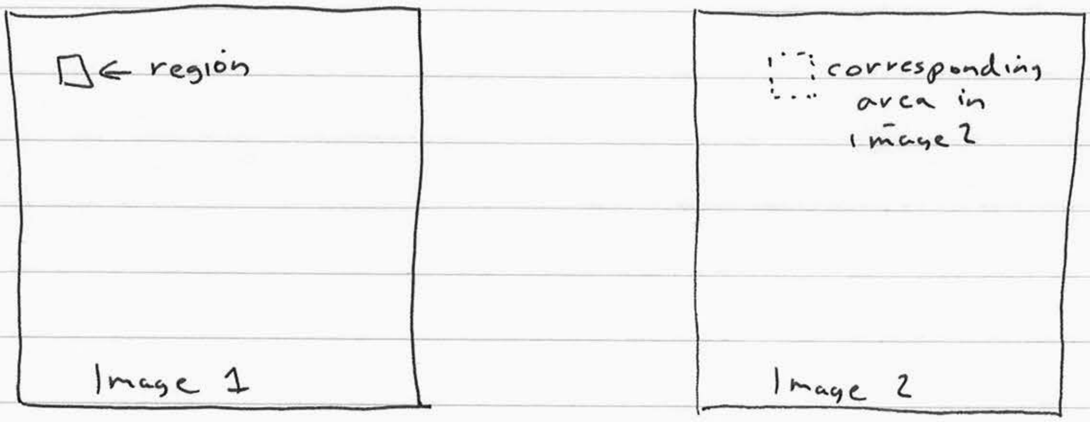
By comparing the positions of like scatterers in both interferometer images, we can derive empirically the relation

$$\delta = B_{\parallel} + B_{\perp} \frac{dr}{r_0 \tan\theta_0}$$

But since we almost always have noisy data this process is imperfect, and a noisy estimate of the offset results. We overcome this by estimating the offsets at a great many locations and solving for δ using least-squares approaches.

There are many ways to determine the offsets, and we'll discuss one here. Since in the general case we can have shifts both in range and azimuth we'll approach this as a 2-D problem.

Begin by identifying a small region within image 1:

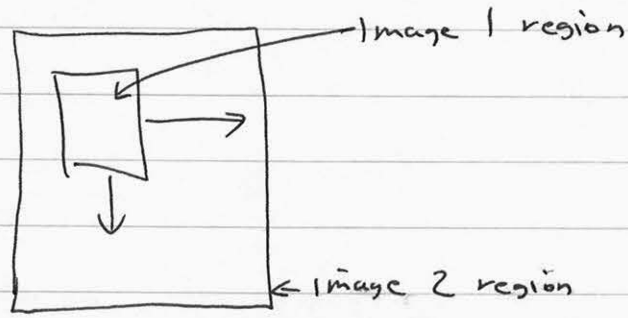


We know approximately, but not exactly, where the region would occur in image 2. We can find the exact location by calculating the cross-correlation

$$\text{region}_{\text{Image 1}} \star\star \, \text{region}_{\text{Image 2}}$$

and determining the cross-correlation peak location. The location of the peak is the offset we seek.

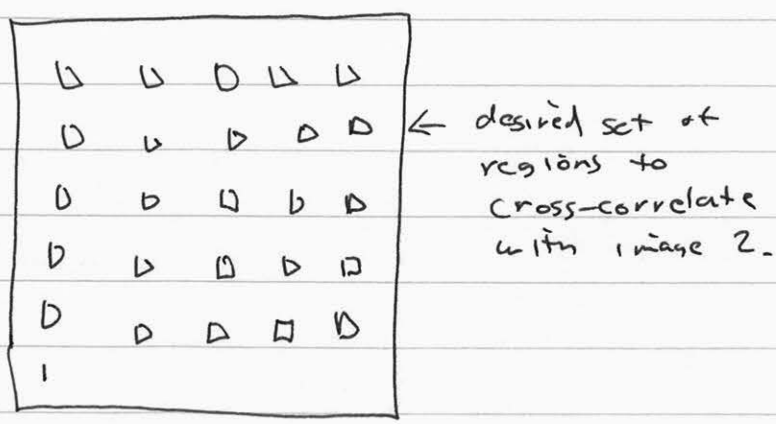
Because we do not know the offsets exactly, we need to choose a larger region in image 2 than in image 1, and then cross-correlate:



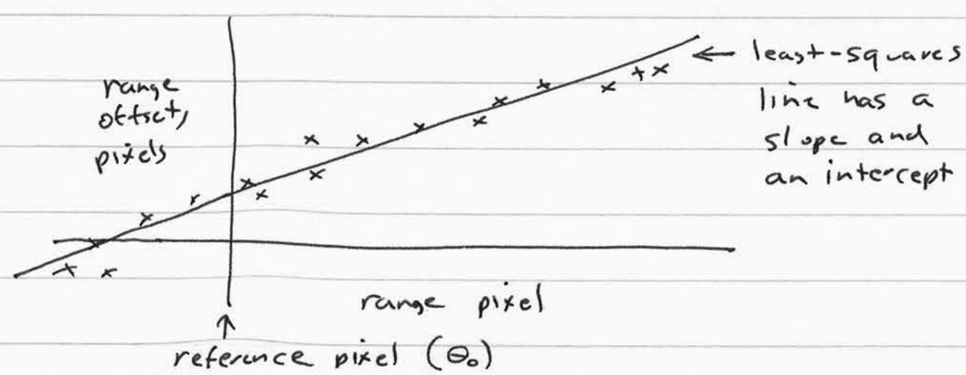
Now, the images are complex quantities when delivered from the processor. To find the peaks, however, we get better performance if we correlate the powers rather than the complex values. This eliminates additional noise that may be due to poor phase fidelity in some regions.

In addition, because we are multiplying two detected images together we effectively have a cross product (in the correlation process) with twice the bandwidth of the original data. This could lead to aliasing if the original data were not oversampled by at least a factor of 2, which is almost never the case. Hence, for best results we would want to interpolate each region by a factor of 2 before the correlation.

Finally, we want to look over the entire image to get the best set of offset estimates. A good rule of thumb is to examine 5-10 locations in range and also in azimuth:



Once we determine all the correlations, we are ready to determine δ empirically. Say we plot the range offset as a function of range pixel number:



Note that the intercept is defined at the reference pixel, that is the one defined by θ_0.

From the measured slope of the line and its intecept, we solve for

$$\delta = B_{\parallel} + B_{\perp} \frac{dr}{r_0 \tan\theta_0}$$

effectively giving us the baseline components $B_{\parallel}$ and $B_{\perp}$.

We can later refine these estimates using known topographic corrections, but for now they are close enough.

The next step is to resample image 2 to the image 1 coordinates using the above relation for δ. In addition if there is an azimuth shift due to timing considerations we would remove that also at this time.

Finally, we form the interferogram itself by computing

$$i(x,y) = \text{image } 1\,(x,y) * \text{conjugate}\left(\text{image } 2\,(x+\delta, y + \text{az-offset})\right)$$

where the complex conjugate multiplication results in a phase differencing operation as desired.

Summary

Our interferogram formation is thus summarized as follows:

1) Process images from each antenna of about the same ground area.
2) Measure the offsets:
   a) Select a small region in image 1 and its

approximate neighbor in image 2.

c) detect complex values to obtain power images of small regions

b) interpolate by 2

d) cross-correlate and record offset peak location

e) repeat a-d for many locations

f) fit line in range dimension to offsets. Also determine azimuth shift, if any.

g) solve for $B_{\parallel}$ and $B_{\perp}$ from line parameters

3) Resample image 2 to image 1 coordinates

4) Cross multiply image 1 by conjugate of image 2.

Multilooking

Once we have the interferogram, we often want to multilook in order to improve our phase estimate accuracy at the expense of spatial resolution. This also reduces the required data volume for storage.

For the image data, recall that we average signal powers rather than complex values. But for the interferogram we need the average phase, so we average in this case the complex values.

In the image case, the phases were uniformly distributed and hence random from point to point, so we averaged power quantities. But for the interferogram the phase changes relatively slowly so coherent integration helps us.

Correlation

Another important measure of interferogram ~~qula~~ quality is the correlation of the two images, or how closely the phase in one image tracks that of the other. This quantity varies from 0 for images completely independent of each other to 1 for identical phase diffences. We can calculate the correlation at many points in an image when we generate the multilooked image, according to the following definition

$$c = \frac{\sum image1_i \, image2_i^*}{\sqrt{\sum image1_i \, image1_i^*} \sqrt{\sum image2_i \, image2_i^*}}$$

where $image1_i$ and $image2_i$ are the complex values of a corresponding point in image 1 and image 2. Note that the numerator is simply the interferogram and the denominator is the product of the image amplitudes, not powers.

We can display correlations as a function of position and can evaluate how the correlation varies as a function of position. We'll discuss this more next time.