Handout # 32

GP265/EE355

Interferometric Techniques and Applications

Many applications of imaging radar involve interpretation of the radar amplitude data using photo-interpretive techniques and the same kinds of scattering models we have just discussed. We'll put these aside for now and concentrate instead on the measured phases of the radar echoes.

Since the phase of an individual radar echo is a statistical quantity that is uniformly distributed over the interval $(0, 2\pi)$ [for reasons we'll discuss below], it is only the phase difference between two measurements that we can readily measure. But measurement of phase differences is a well-developed concept in optics, denoted interferometry, and we can apply the same approaches to radar signals. This will allow us to measure surface shapes, or topography, as well as surface motions, useful for a great many geophysical investigations.

The radar distance measurement

When we measure a radar echo and form a radar image, we generate a high-resolution map of the 2-D brightness distribution of a surface. This distribution forms a "picture" of the terrain which we can interpret.

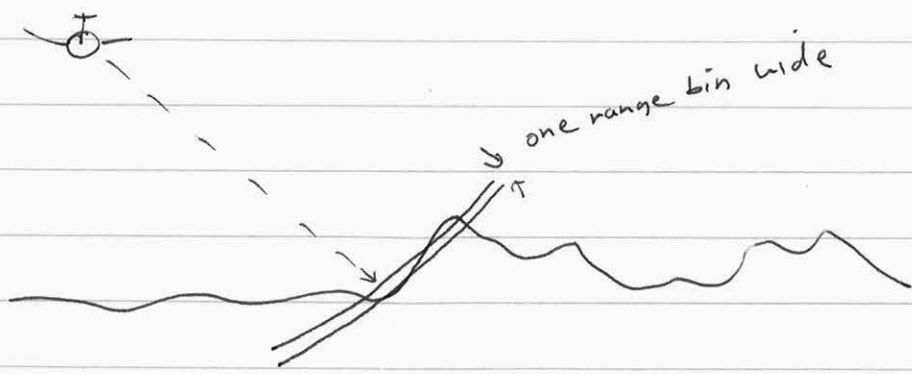
To form the image, say using the SAR algorithm, recall that we not only focus the image in the two dimensions but we remove any phase effects that follow from platform motion. Hence our image has a time delay associated with its distance (let's assume we use r_0 as a reference) but also the constant phase

$$\phi = \frac{-4\pi}{\lambda} r_0$$

Each pixel thus carries with it phase values that are related to the distance of that pixel from the radar.

Interferometry for topography

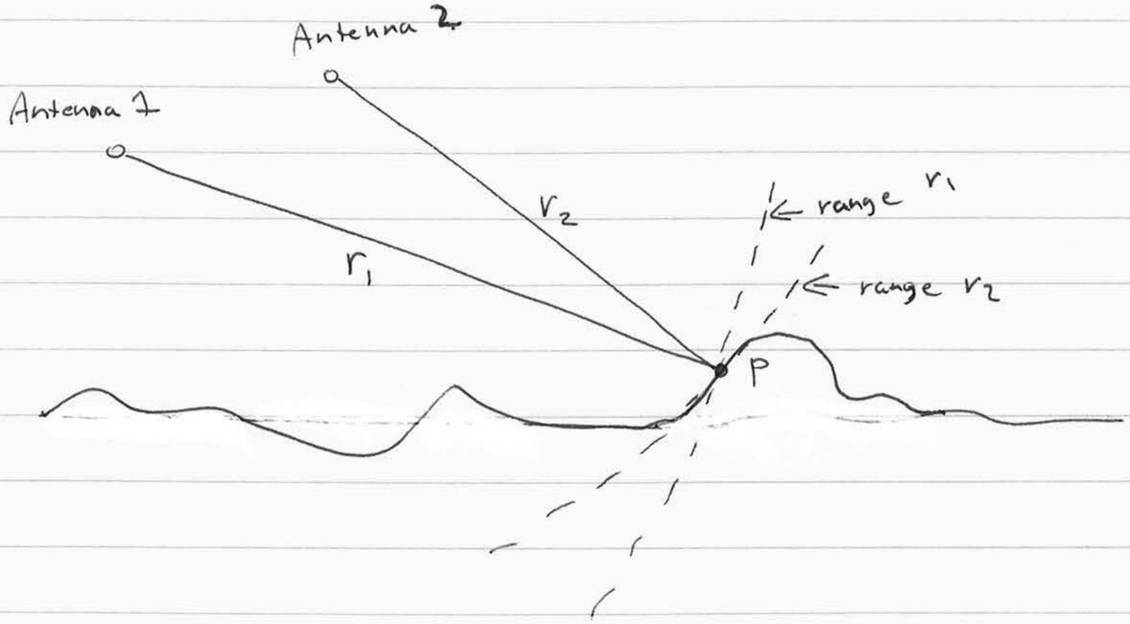
Consider imaging the following scene from a radar flying into the page:



Note that the return gives contributions from all points within a band the width of a range resolution element at the same time. Another way of thinking about this is that any scatterer located within that band would have the same time, Doppler, and phase history. In other words the radar is not at all sensitive to surface heights. Thus the radar image is two-dimensional in nature, and we can only infer topography from perceived changes in brightness.

Incidentally, ~~this~~ such a method of inferring topography from brightness is often used in radar image (or optical image) interpretation, and is called "shape from shading".

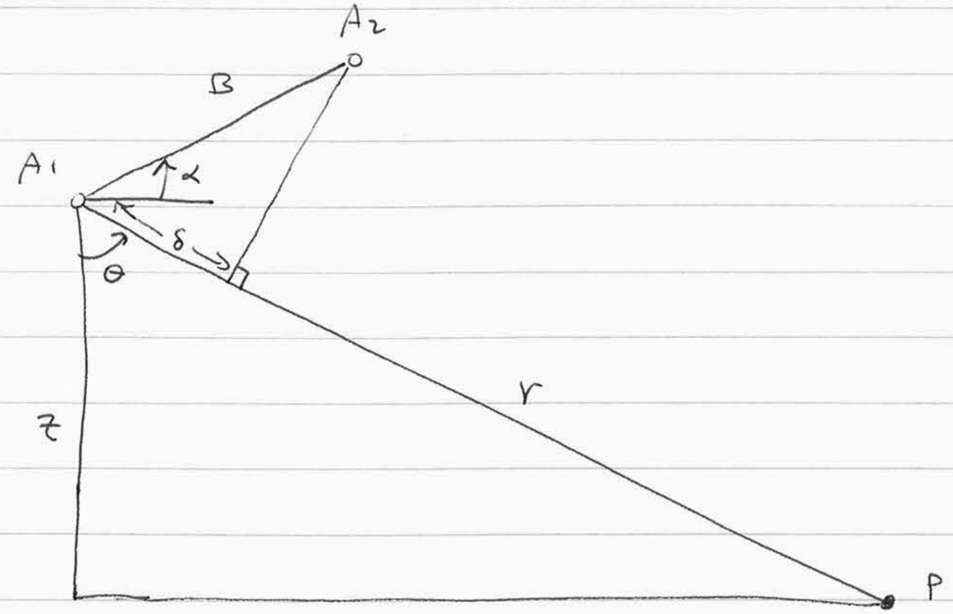
Now, let's change the geometry of the situation and add a second radar antenna. What are the distance measurements made by each to a point on the terrain?



The distance measurement r_1 from antenna 1 and r_2 from antenna 2 define two circles which intersect at the point P. If we could measure each then we could determine the 3-D coordinates of P.

If r_1 and r_2 are quite different, we can use this method of radar parallax (or radar stereo) for topographic mapping. But for $r_1 \approx r_2$ the method is not very sensitive to topography, and other considerations lead to operational difficulties.

Instead, consider the following implementation where we have made the assumption that $|r_1 - r_2| \ll |r_1|$, the so-called parallel-ray approximation:



Define a "baseline" B between the two antennas. Signals transmitted from A1 and received back at A1 travel a distance 2δ further than those transmitted and received at A2. (If we compare propagation lengths A1 – P – A1 and A1 – P – A2 the difference is only δ.)

Denoting the orientation of the ~~the~~ baseline with respect to horizontal as α, we have:

$$\delta = B \sin(\theta - \alpha)$$

and the height z

$$z = r \cos\theta$$

Thus if we can measure δ we can infer the topographic height z. Now, how accurately can we do this? We need to evaluate the derivative

$$\frac{\partial z}{\partial \delta} = \frac{\partial z}{\partial \theta} \cdot \frac{\partial \theta}{\partial \delta}$$

We'll find it easier to evaluate the derivative in stages like this.

$$\frac{\partial z}{\partial \theta} = -r \sin\theta$$

$$\frac{\partial \delta}{\partial \theta} = B\cos(\theta - \alpha)$$

so $$\frac{\partial z}{\partial \delta} = \frac{-r\sin\theta}{B\cos(\theta-\alpha)}$$, and thus

$$\sigma_z = \frac{r\sin\theta}{B\cos(\theta-\alpha)}\,\sigma_\delta$$

Therefore, each error of one meter in δ leads to an error in z of about $\frac{r}{B}$ m, which can be large. Given the precision in range of one resolution element δ_r, the error in topography is about

$$\sigma_z = \frac{r}{B}\,\delta_r$$

which is why we would want to keep B very large.

But instead of comparing lengths by differencing ranges, let's use our knowledge of the phase of the returns. Then, if we were to compare phases we'd get the following additional relation:

$$\phi_1 - \phi_2 = \frac{-4\pi}{\lambda}\left(|r_1| - |r_2|\right)$$

or

$$\phi = \frac{-4\pi}{\lambda}(r_1 - r_2) = \frac{-4\pi}{\lambda}\,\delta$$

Now, we can typically measure phase to a few degrees or so, equivalent to measuring δ to about $\frac{1}{100}$ of a wavelength. Then our relation for accuracy is about

$$\sigma_z = \frac{r}{B} \cdot \frac{\lambda}{100}$$

Even if $\frac{r}{B}$ is large, $\frac{\lambda}{100}$ is small so we get reasonable performance.
Look at an ERS example:

$r = 800$ km

$B = 1000$ m

$\lambda = 6$ cm

$\delta_r = 9$ m

Stereo case: $\sigma_z \approx \frac{r}{B} \delta_r \approx 7200$ m ← Because of small B

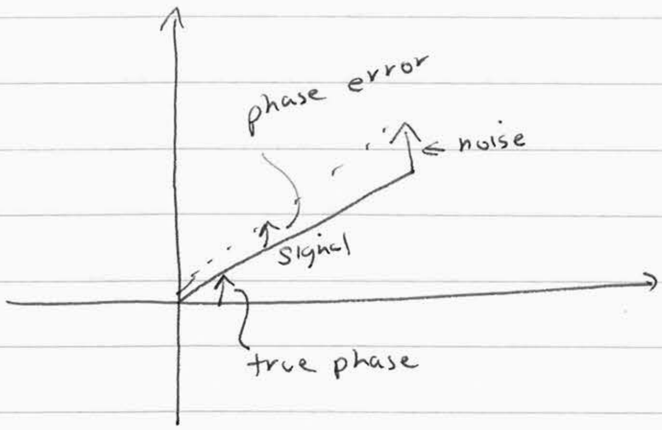
Interferometer case: $\sigma_z \approx \frac{r}{B} \frac{\lambda}{100} = \frac{1}{2}$ m

These are extreme examples but you see the power of the interferometric approach.

Accuracy of phase measurements

Clearly our interferometric performance depends critically on how well we can measure phase, which in turn depends on signal to noise ratios. We know how to estimate SNRs, so how does that map into phase accuracy?

Let's model an echo signal as consisting of a "signal" vector with unit length and a "noise" vector added to it: (These are both complex quantities)



Very approximately the phase error $\Delta\phi$ will be $\dfrac{|\text{noise}|}{|\text{signal}|}$ if the noise power is much less than the signal power. The variance of the noise σ_ϕ^2 will then be about

$$\sigma_\phi^2 \approx \frac{|\text{noise}|^2}{|\text{signal}|^2} = \frac{1}{SNR}$$

So, σ_ϕ in radians is about $\dfrac{1}{\sqrt{SNR}}$. If the SNR is 100, or 20 dB, $\sigma_\phi = 0.1$ radian $\approx 6°$.

Collecting much of the above together, we can summarize the above error analysis as:

$$\sigma_z = \frac{\lambda}{4\pi} \cdot \frac{r \sin\theta}{B\cos(\theta-\alpha)} \cdot \frac{1}{\sqrt{SNR}} \quad \leftarrow \text{valid for high SNR}$$

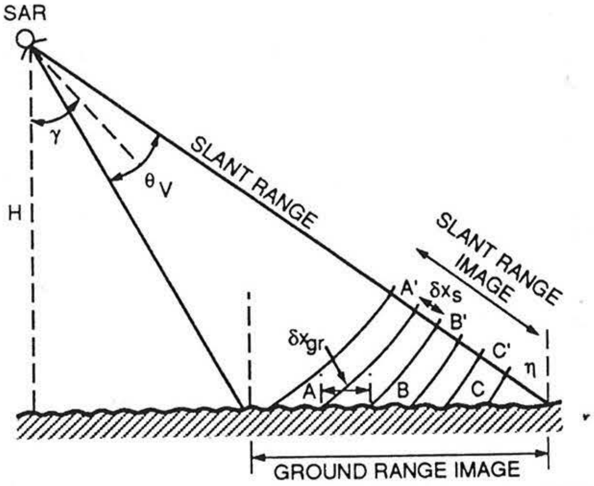
<u>Cartographic applications</u>

Once we have the interferometric technique available how can we use the data in cartographic studies? Let's briefly talk about geocoding and georeferencing, and then spend some time looking at several actual applications.

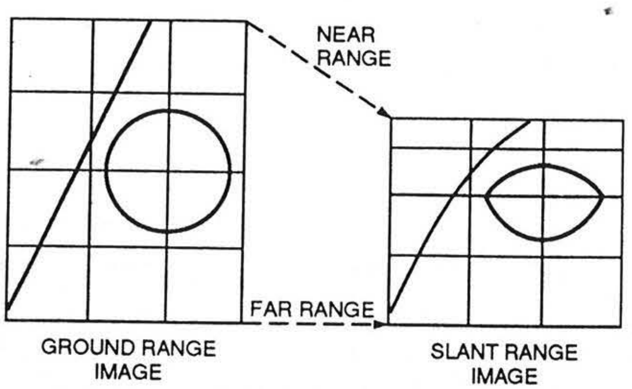
Geocoding and georeferencing (see chapter 8 of the text)

A typical SAR image is presented in a coordinate system where the range coordinate is slant range and azimuth is correct along-track distance. This leads to foreshortening or even layover in the resulting image. The issue is the nonlinear mapping of slant range into ground range, plus uncompensated topography.

Here are some illustrations of geometrical distortions: (from the text)



Definition of Slant-to-Ground Range Quantities



← note: should have "smooth" sides

Figure 8.5 Relationship between slant range and ground range image presentation for a side looking radar.

Foreshortening:

Extreme Foreshortening

(Layover)

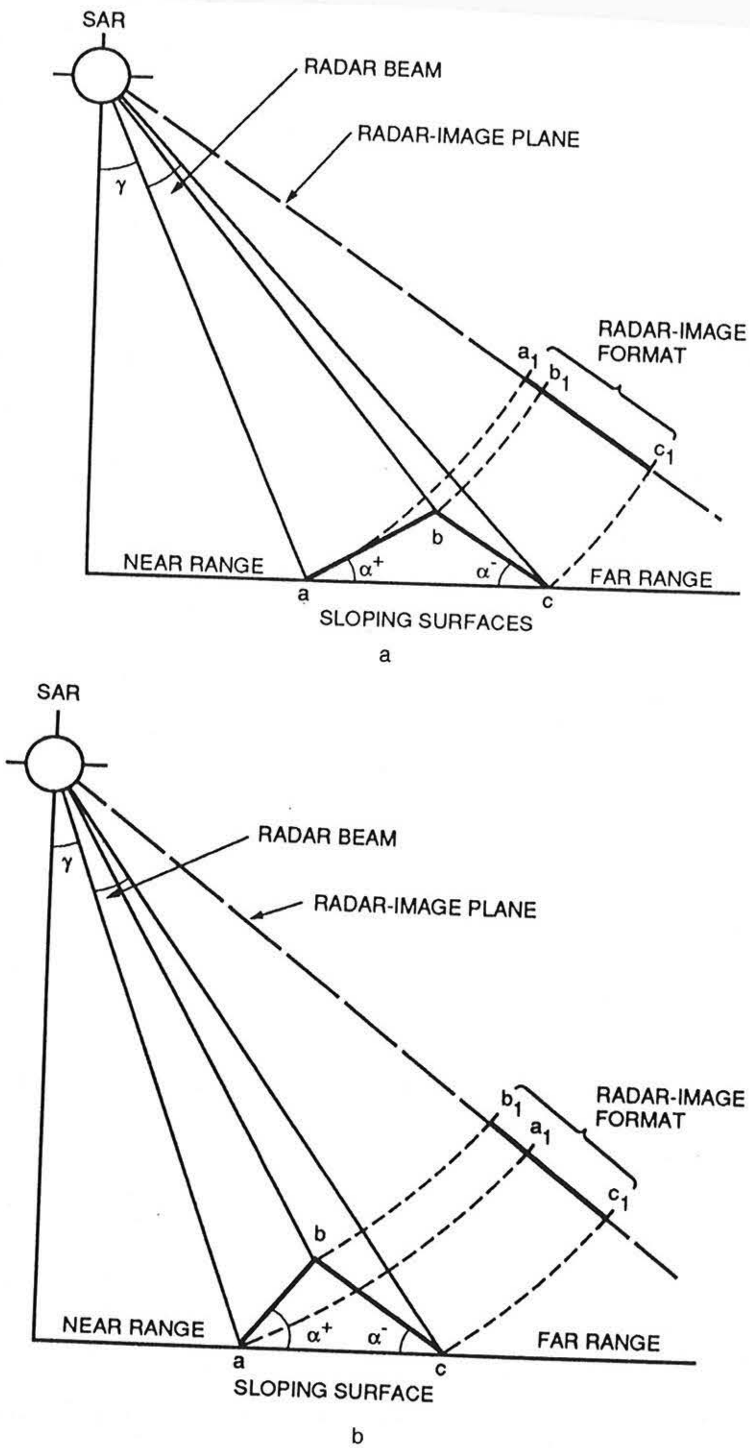
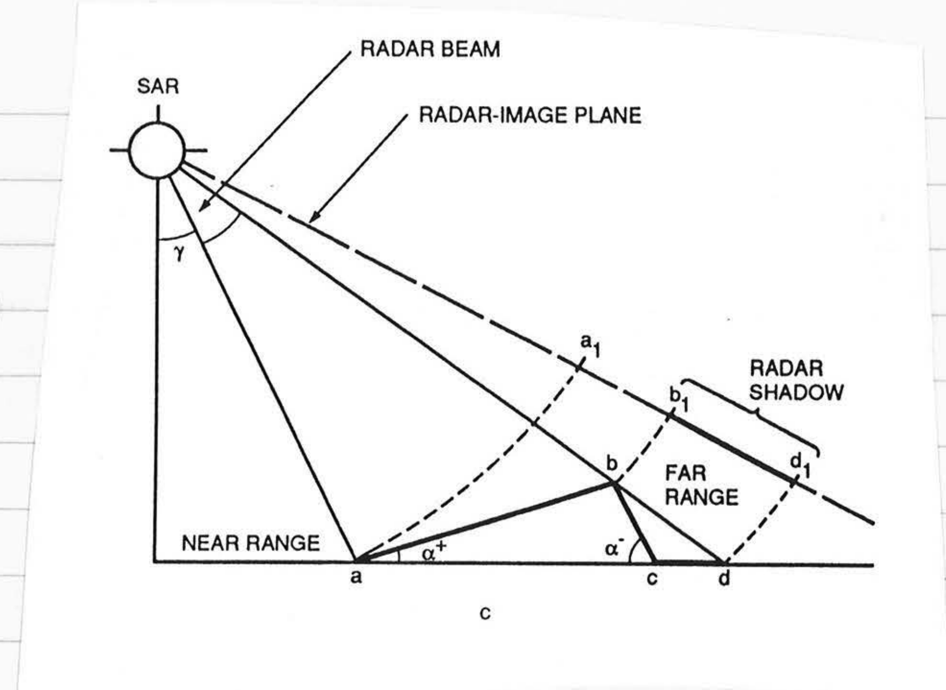
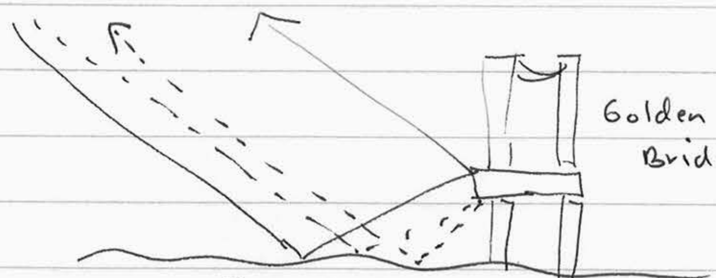


Figure 8.6 Geometric distortions in SAR imagery: (a) Foreshortening; (b) Layover; (c) Shadow; (d) A combination of imaging geometries illustrating secondary peak.

Shadowing



Multipath



Knowledge of the heights of the terrain allows us to correct for each of these effects. We know that

$$r_{ground}^2 + z^2 = r_{slant}^2$$

so if we resample accorinding to

$$r_{ground} = \sqrt{r_{slant}^2 - z^2}$$

we'll get a "correct" image, at least as far as we can manage. Shadows and extreme foreshortening still provide distortions however.