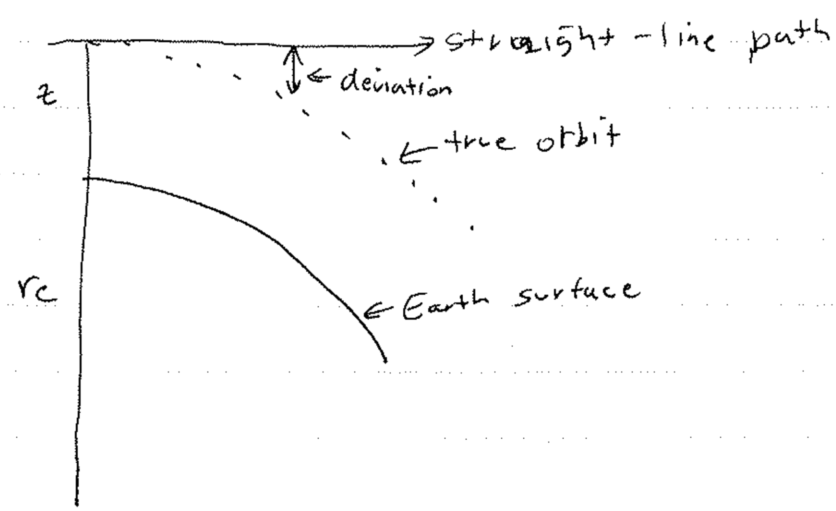
Setting SAR processing parameters

When it comes down to finally making SAR images from radar data, we must choose the correct set of processing parameters based on the geometry of the imaging system. As when we discussed solving for the Doppler centroid, often we don't know the geometry in a precise enough form to completely focus the image. We thus must solve for one parameter or another from the data themselves.

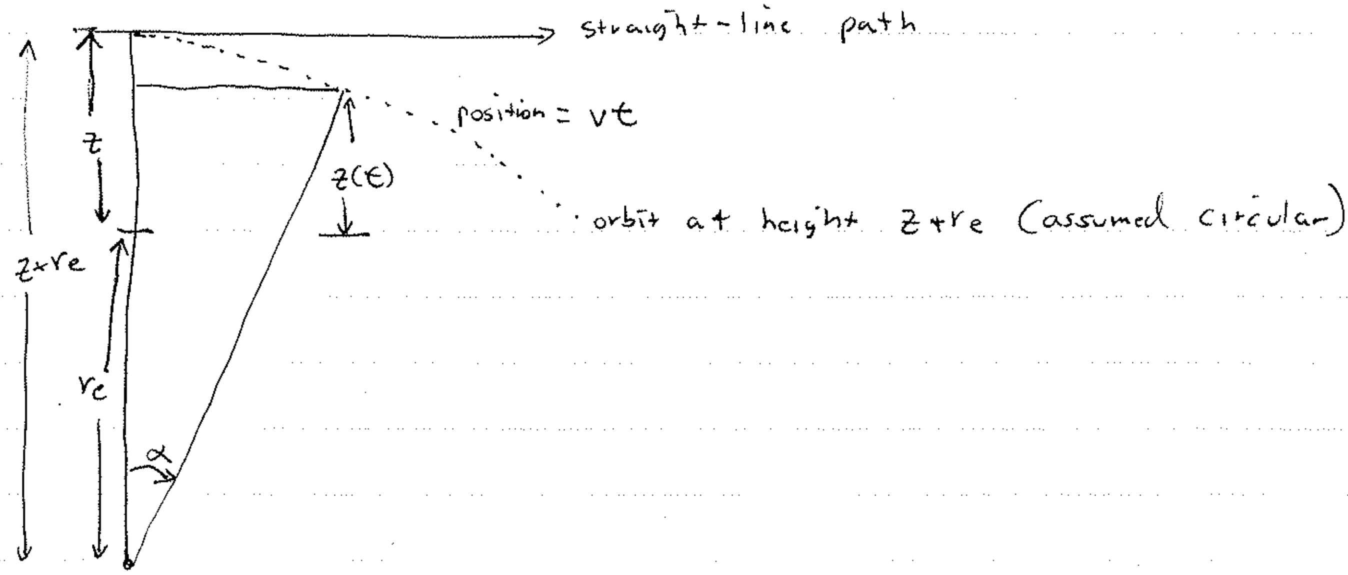
Also, whether or not we are in an aircraft or spacecraft platform makes a difference. We'll look at this issue first.

Effective velocity for spacecraft

Our previous solution for chirp rate f_R ~~[illegible]~~ was based on an assumption of straight-line motion of the platform. But in an orbiting radar the orbits are not straight, rather they are ellipses centered near the Earth center. This curve deviates from the assumed straight-line path, giving an effective height z that changes with time:



We can find an effective velocity that will still focus the image as follows. Use this construction:



The height $z(t)$ thus effectively decreases with time away from the reference zenith point. We have

$$\sin\alpha = \frac{vt}{z+r_e}$$

and

$$r_e + z(t) = (r_e + z)\cos\alpha$$

so

$$z(t) = (r_e + z)\cos\alpha - r_e$$

For small α, which we always have,

$$z(t) = (r_e + z)\left(1 - \frac{v^2t^2}{2(z+r_e)^2}\right) - r_e$$

$$= z - \frac{1}{2}\frac{v^2t^2}{z+r_e}$$

Now, our usual expression for $r(t)$:

$$r^2(t) = z^2(t) + y^2 + v^2t^2$$

$$= \left(z - \frac{1}{2}\frac{v^2t^2}{z+r_e}\right)^2 + y^2 + v^2t^2$$

$$\approx z^2 - \frac{z\,v^2t^2}{(z+r_e)} + y^2 + v^2t^2 \qquad \left(\text{neglecting } \frac{v^4t^4}{(z+r_e)^2} \text{ term}\right)$$

$$= z^2 + y^2 + v^2t^2\left(1 - \frac{z}{z+r_e}\right)$$

This is the usual result, but with an effective velocity v_{eff}:

$$v_{eff}^2 = v^2\left(1 - \frac{z}{z+r_e}\right)$$

$$= v^2\left(\frac{r_e}{z+r_e}\right)$$

or $$v_{eff} = v\sqrt{\frac{r_e}{z+r_e}}$$

Example: To focus a satellite in an 800 km orbit with $v = 7500$ m/s,

$$v_{eff} = 7500 \cdot \sqrt{\frac{6378}{800+6378}} = 7070 \text{ m/s}$$

Is this effect important for aircraft? ($v = 200$ m/s, $z = 8$ km)

$$v_{eff} = 200\sqrt{\frac{6378}{8+6378}} = 199.875 \text{ m/s}$$ ← only for the most precise applications!

Azimuth focus parameters

Azimuth focusing errors are evident in SAR images if the chirp rate f_R is ~~not~~ accurately known. How accurately do we need to know this rate? As before, a good rule of thumb is to one part in the time-bandwidth product. Let's see what this means for a airborne and spaceborne L-band radar.

	λ	r	v	ℓ	τ_{az}	Bw_{az}	$\tau \cdot Bw$
A/C	.24	15×10^3	200	1	18	400	7200
S/C	.24	850×10^3	7500	10	2.72	1500	4080
					$= \frac{r\lambda}{v\ell}$	$= \frac{2v}{\lambda}\cdot\frac{r\lambda}{\ell r}$	$= \frac{2r\lambda}{\ell^2}$

So the ~~rate~~ chirp rate must be known accurately indeed. Usually measurements aren't this accurate, so we often need to estimate the chirp rate from the data. We will look at two algorithms to do this, the maximum variance method and the sub-aperture shift method.

Maximum variance. This method involves calculating images using several trial values for f_R, and choosing the value that maximizes the image power variance. The advantages of this approach ~~are the~~ is that it is easy to compute.

Consider what happens when we vary the focus of an image. A blurred image has a more uniform distribution of energy (power):

Well-focused image

Misfocused image

Note that the poorly focused image has less contrast than the crisply focused image. Hence the variance of the focused image is higher than the blurred image. If we were to compute the variance of the image as a function of f_R, we could choose the value of s that maximizes the quantity

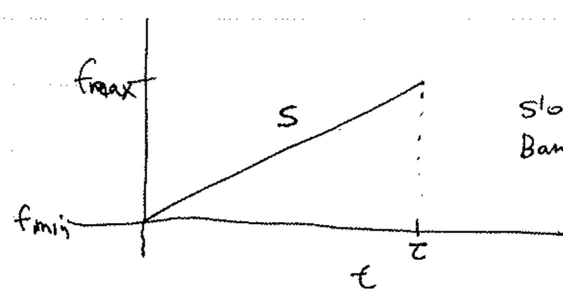
$$\sigma^2_{power} = \frac{1}{N}\sum_{j=1}^{N} |i_j(t)|^4 - \left(\frac{1}{N}\sum_{j=1}^{N} |i_j|^2\right)^2$$

where i_j is the jth sample of the image sequence $i_1, i_2, \cdots i_N$. The N samples can be distributed two-dimensionally, of course.

Now, since misfocus redistributes energy but conserves it if the reference function is appropriately normalized, in practice we can ignore the second term above and simply maxime the second moment of the image power $|i_j|^2$.

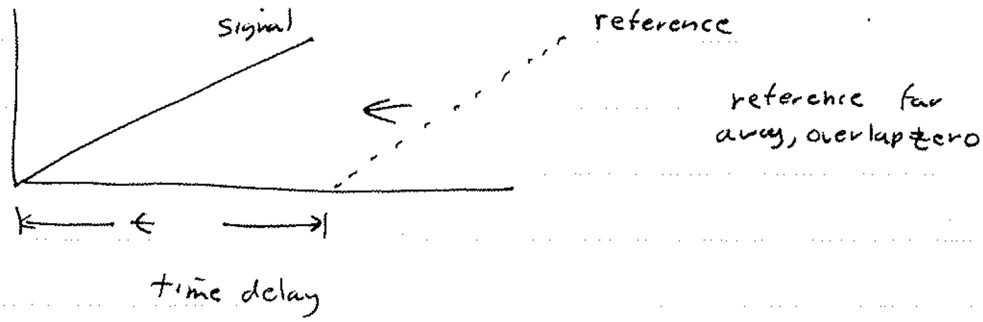
Sub-aperture shift. Another approach examines the shift of images formed from sub-apertures of the full synthetic array. To understand this method we must first introduce a tool for graphical analysis of chirp processing, frequency/time plots.

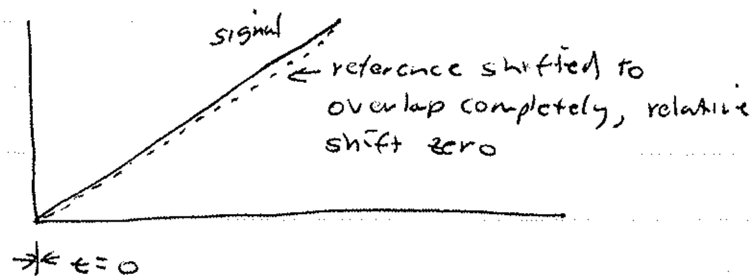
Frequency-time plots, or spectrograms, of the range or azimuth modulation allow us to visualize the correlation process and some of the properties of compressed signals. Consider first a graph of frequency vs. time for a chirp from f_1 to f_2 over a time τ:



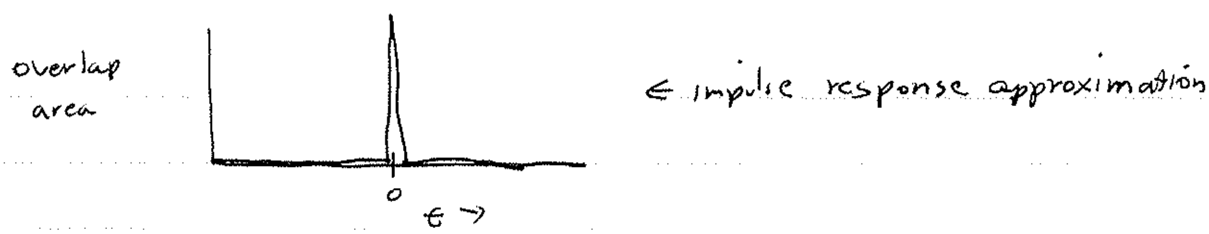
Slope of chirp is s
Bandwidth is $f_2 - f_1$

Next, ~~convolve~~ correlate this signal with itself by sliding a copy of it by and plotting the area of overlap. Assume some finite width to the line for the purposes of this exercise.

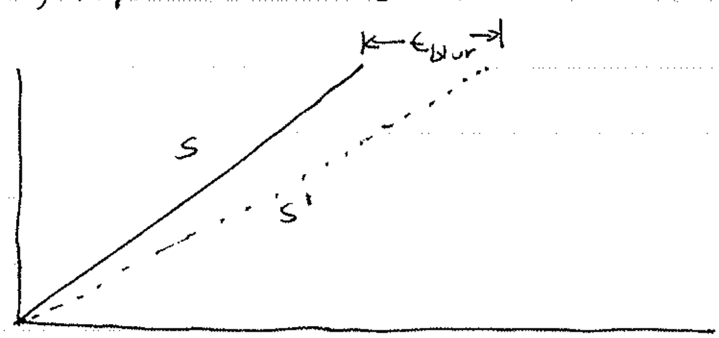




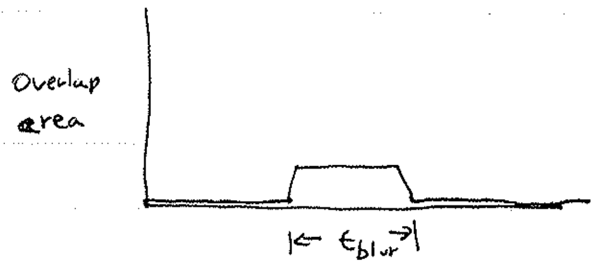
If we plot the overlap area vs. the shift ϵ, we get a near-δ response:



Now, repeat the exercise with the reference chirp having a slope $s' \neq s$.

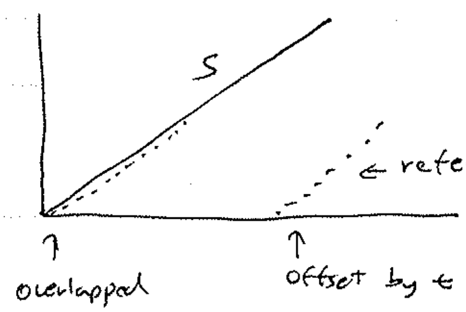


The output is blurred over a time t_{blur} as follows:

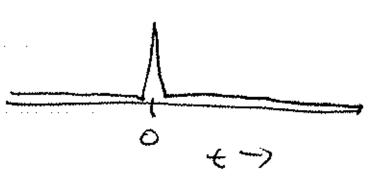


The output is lower and defocused, as would be expected from using the incorrect value of s' in the reference.

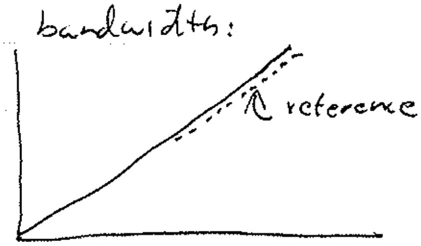
Sub-aperture processing. The next step is to consider what happens if we use the correct chirp slope s but process only part, in this case half, of the available bandwidth:



This plots out to the impulse response, but the width is doubled due to half the bandwidth being used:

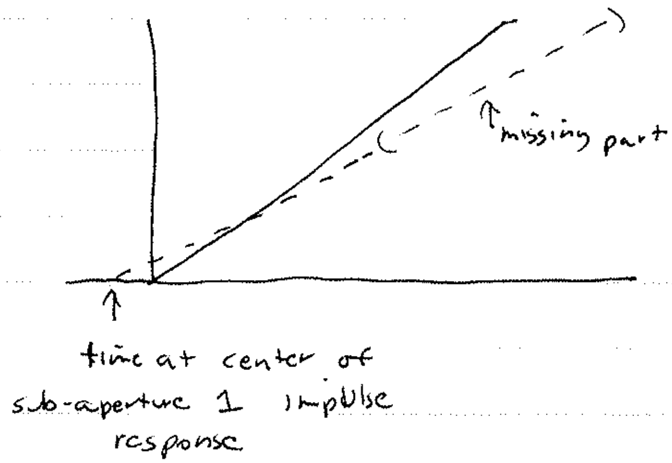


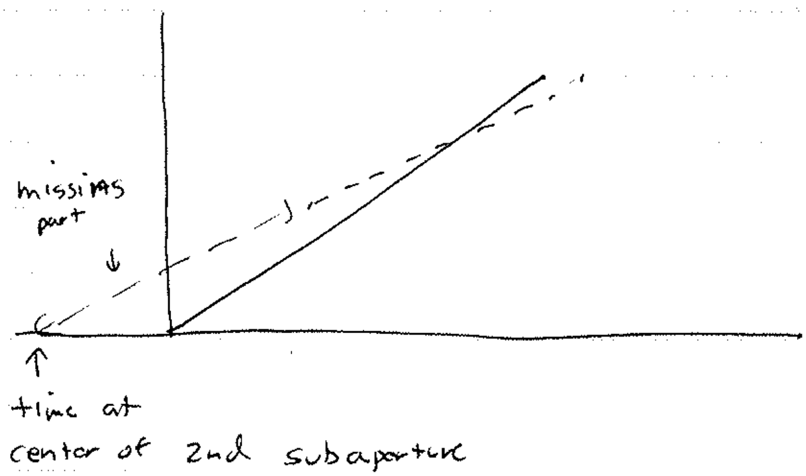
Compare this with the result from a sub-aperture using the other half of the bandwidth:



Note that the response from the second aperture is identical to that from the first subaperture. This in fact forms two separate "looks" of the same data -- this implementation is called "taking looks in the frequency domain", or "frequency-domain subaperture processing".

Now, what happens if we generate the subapertures from a reference whose slope is not the same as the signal? The output images are no longer aligned in time.

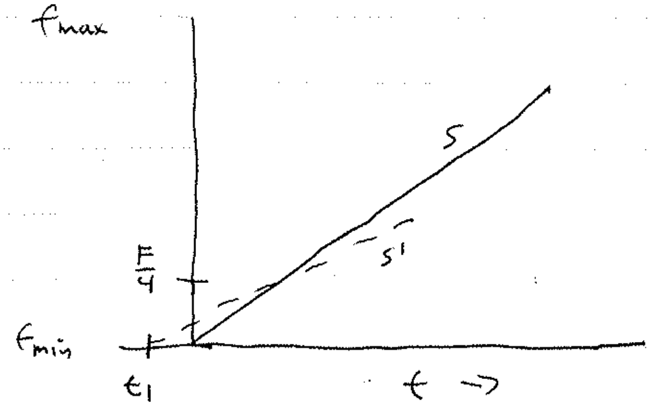




Of course the response in each channel will be broadened due to the misfocus, but also the two images will be shifted with respect to each other.

The amount of shift is proportional to the error in chirp rate, $s'-s$. We see that by the following construction.

Let's draw the plot at the point of maximum output for aperture 1.



$f(t) = s \cdot t + f_{min}$

$F = f_{max} - f_{min} =$ Bandwidth

The maximum output time t_1 is found from the construction to be

$$t_1 = \frac{F}{4s} - \frac{F}{4s'}$$

Similary the peak output for the second subaperture t_2 is

$$t_2 = \frac{3F}{4s} - \frac{3F}{4s'}$$

Hence the shift $\Delta t =$

$$\Delta t = t_1 - t_2$$

$$= \left(\frac{F}{4s} - \frac{F}{4s'}\right) - \left(\frac{3F}{4s} - \frac{3F}{4s'}\right)$$

$$\Delta t = \frac{F}{2s'} - \frac{F}{2s}$$

$$= \frac{F}{2}\left(\frac{1}{s'} - \frac{1}{s}\right)$$

$$= \frac{F}{2}\left(\frac{s - s'}{ss'}\right)$$

or,

$$\Delta t \approx \frac{F}{2}\left(\frac{\Delta s}{s^2}\right) \qquad (\text{for } s' \approx s, \text{ and } \Delta s = s - s')$$

Inverting this, we can obtain the error in slope from the offset:

$$\frac{2 \Delta t \, s^2}{F} = \Delta s$$

So, if we measure the image offset Δt we can obtain a correction to the slope Δs, and iterate until we have good focus.

⟹ We obtain the offset Δt by correlating the two sub-aperture images against each other.

So we can obtain optimal focus by subaperture processing, cross-correlation, and slope correction. In practice, we process a full patch from each subaperture, correlate ~~line by line in az~~ line by line in azimuth and average the correlations.

Note that we solve for the chirp rate (or slope) by either of these methods. But, in azimuth the rate f_R is

$$f_R = \frac{-2v^2}{\lambda r}$$

So we can interpret an error in slope as either an error in velocity or in range, since we usually know the wavelength. Either of these may be known from other means, permitting solution of the other, or else we must be content with knowing only the ratio $\frac{v^2}{r}$.