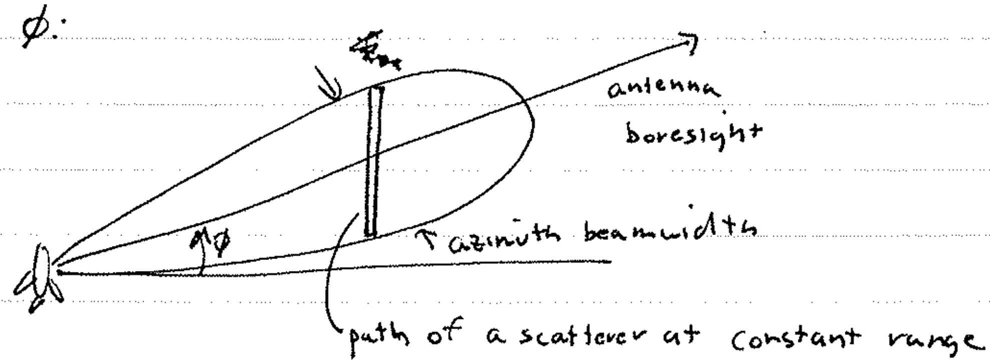
The Doppler centroid

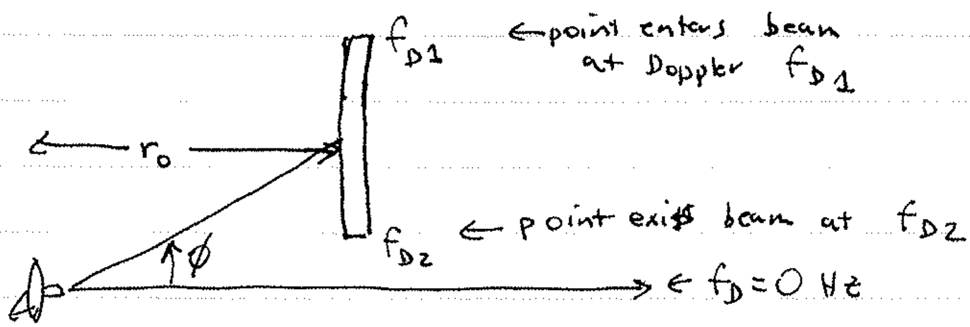
In many instances the radar antenna is not pointed exactly broadside to the line of flight, in fact in many cases we don't know where the beam is pointed. In other words, it has some unknown squint angle. Since we must know our geometry in order to place our echoes at the correct location in the swath, and also to process the data accurately, we desire techniques to determine the antenna pointing.

Sometimes we can solve for antenna pointing from detailed measurements made by the inertial navigation system on the plane or spacecraft. However, we can also solve for the pointing from the data themselves.

Consider an antenna squinted forward some unknown angle ϕ:



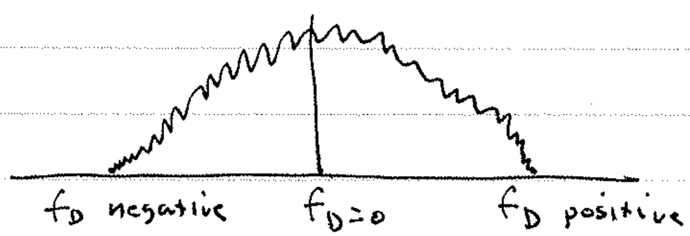
Consider in more detail the path traveled by a scatterer as the plane flies by with this skewed geometry:



Thus at any instant in time the return in range bin r_0 contains scatterers ranging in Doppler frequency from f_{D2} to f_{D1}, which may or may not contain the broadside point at $f_D = 0$.

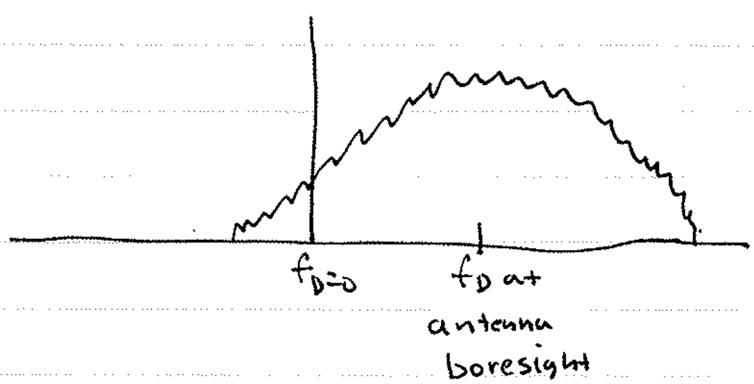
Suppose also that the reflectivity is uniform over the illuminated strip. Then we could plot a graph of the received Doppler spectrum.

Case 1. No squint angle



Note the peak at $f_D=0$ as the antenna boresight is set at 0 radians. As we examine other scatterers the total power falls off as we appoach the edges of the antenna beam.

Case 2. Positive squint



In this case the energy peaks up at a positive value of f_D. If we measure the centroid of this curve, we can solve for the squint angle by our usual Doppler equation:

$$f_{D_{peak}} = \frac{2v}{\lambda} \sin\theta \sin\phi$$

or

$$\sin\phi = \frac{\lambda f_{D\,peak}}{2v \sin\theta}$$

Practical calculation of f_D

Clearly a simple method of estimating f_D is to simply plot a spectrum of Doppler frequency and numerically evaluate its centroid. However several items make this harder than it sounds:

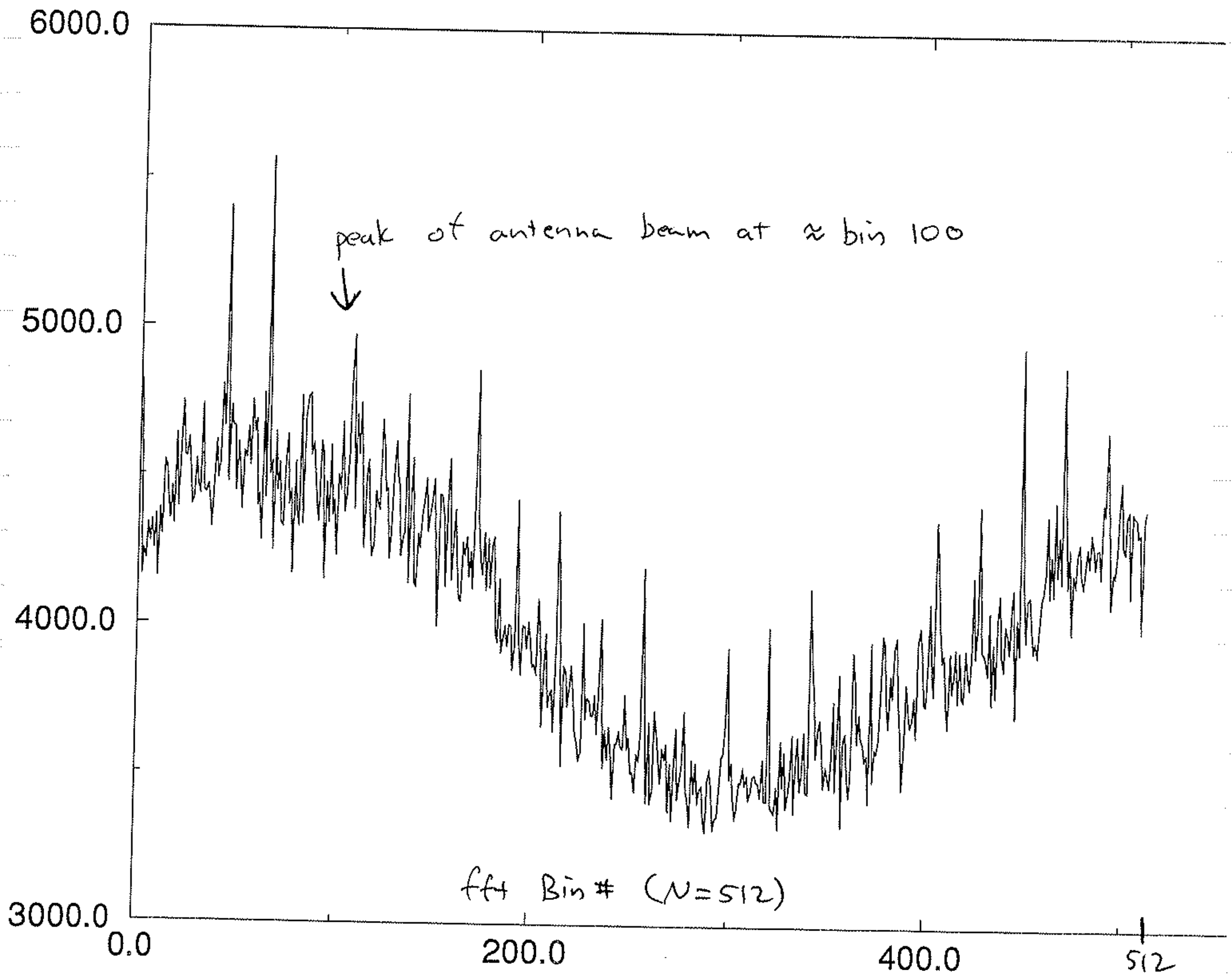
1) The reflectivity ~~map~~ may not be constant, i.e., there may be bright spots in the image.

We typically deal with this problem by averaging the Doppler spectra incoherently over, say, 100 successive range bins. Variations in backscatter will tend to cancel out, and we use the approximate range at the center of the averaged interval in our centroid to squint calculation.

2) The prf may be too low to completely eliminate wrap-around of the spectra in the fft.

This causes it to be more difficult to find the spectral centroid. Because the signal is noisy some interpretation by eye is ~~occasion~~ occasionally required, but clever algorithms can work.

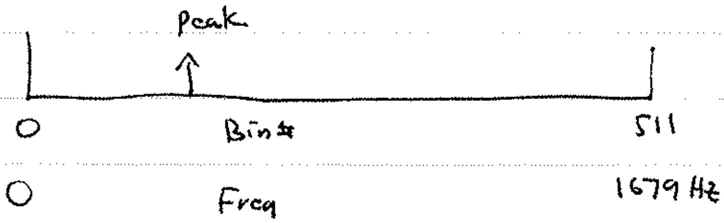
For example, here is a picture of a spectrum derived from the ERS-1 satellite:



Note the curve is noisy and has kinks in it denoting changes in reflectivity. This is an average of 512 spectra.

Suppose the peak is truly in bin # ~~512~~ 100. What is the doppler centroid of these data? How about the squint angle?

For that you have to know the prf of the radar, which in this case is 1679 Hz. Then consider the frequency assignments in the fft:



The ~~freq~~ total frequency span is 1679 Hz, so $\Delta f = 3.28$ Hz/bin and the centroid would be 328 Hz.

For the squint angle, you need some additional info:

~~range = 850 km~~

$\theta = 23^\circ$

$v = 7500$ m/s

$\lambda = 6$ cm

then $$328 \text{ Hz} = \frac{2 \cdot 7500 \text{ m/s}}{6 \text{ cm}} \cdot \sin 23^\circ \cdot \sin \phi$$

or $$\sin \phi = \frac{328 \cdot 0.06}{2 \cdot 7500 \cdot \sin 23^\circ} = 3 \times 10^{-3}$$

$$\Rightarrow \phi \approx 0.19^\circ$$

<u>Another centroid algorithm – average phase shift</u>

Another algorithm we can use for centroid estimation is to evaluate the average phase shift from line to line in azimuth. Since the largest signals will, on average, be those from the center of the antenna beam, suppose we determine the ~~one~~ phase shift at each range bin from line to line in azimuth, and weight each estimate by amplitude. Then after enough averaging the result will be dependent on the antenna beam pointing.

We can do this by the following recipe. Suppose we have an image that has been compressed in range. For each line, sum the product of each line with the conjugate of the previous line, and accumulate the complex sum:

$$\text{result at bin } i = \sum_{line=2}^{nlines} r(i, line) * \text{conjg}(r(i, line-1))$$

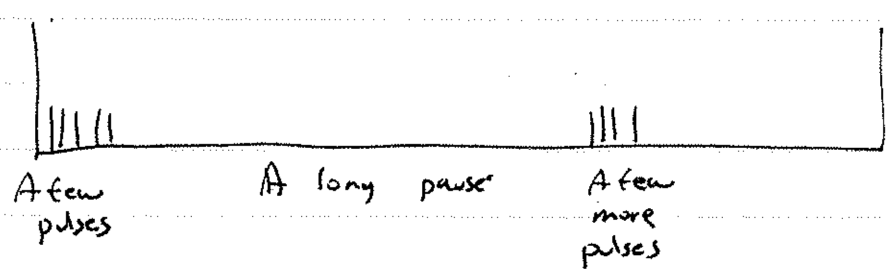
This then gives the average complex change from line to line. If we look at the argument of the ith result

$$\arg(\text{result}_i) = \tan^{-1} \frac{\text{imag part}(\text{result } i)}{\text{real part}(\text{result } i)}$$

a phase change of 2π would correspond to 1 prf in frequency, π would correspond to $\frac{prf}{2}$, etc.

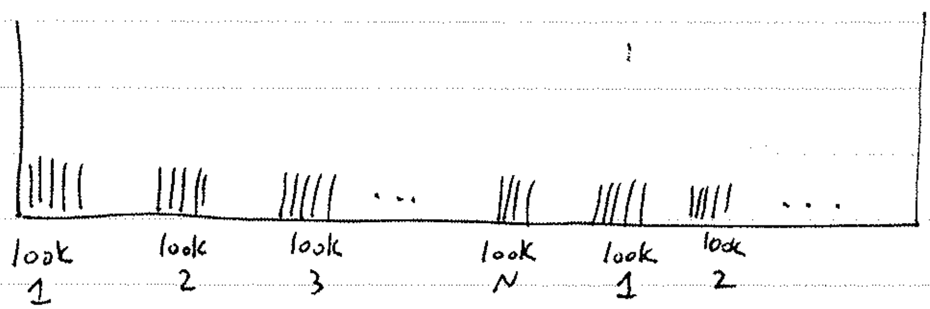
<u>Looks and a practical unfocused processor</u>

We saw previously that to form the unfocussed image we needed to use only a small amount of the data. We had:



The output from such a processor will be a reflection of the true reflectivity, but it will be very "noisy" because of the statistics of the radar image. We'll be covering this more in detail later, but for now suffice it to say that the radar backscatter value we measure represents a single draw from a Rayleigh (for the amplitude) or exponential (for the power) distribution. We overcome this by evaluating several "looks", or independent estimates of the image, and averaging them.

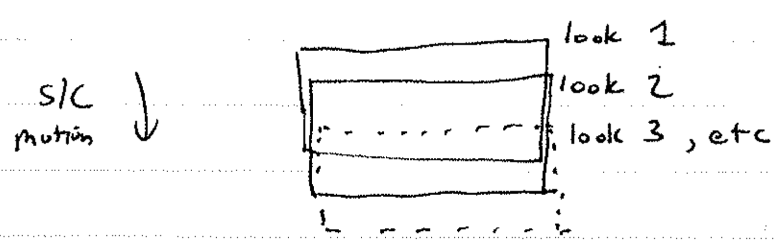
So we might acquire data like this:



where we repeat the look index after we have flown one complete antenna beamwidth.

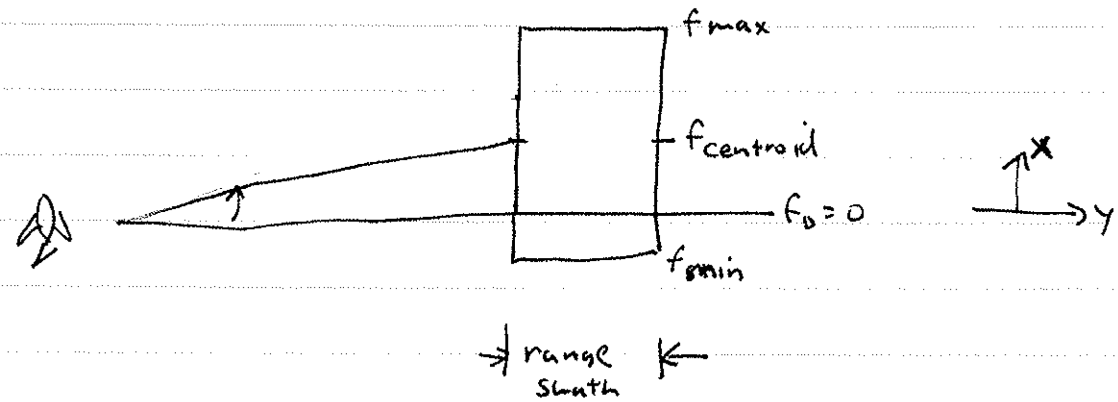
But, note that look 1 and look 2, say, at the beginning of the array, do not illuminate exactly the same piece of ground.

Why? Because the platform is in motion, and therefore the illuminated area moves forward. So we cannot average the looks directly, but must offset each by a few pixels to compensate for the motion, as in the following



Relationship of centroid to area imaged

Consider a patch illuminated by a squinted radar, and its range of Doppler frequencies:

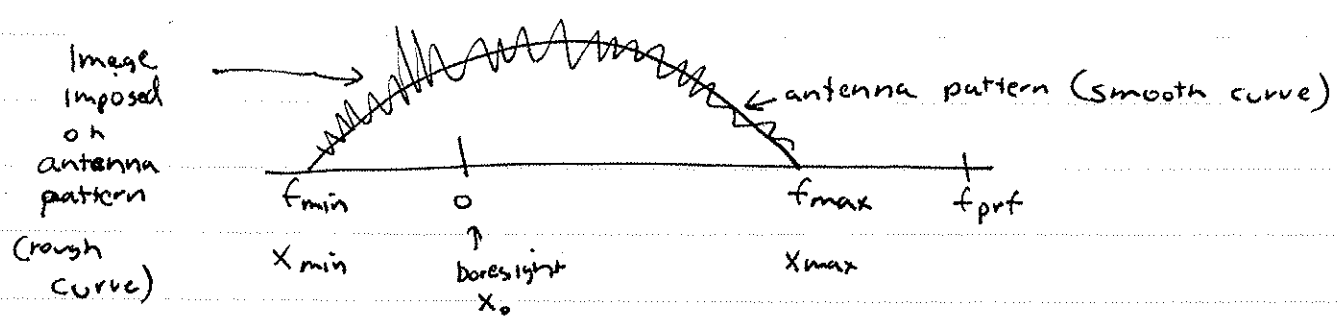


Note here we use the composite squint angle sq ($\sin sq = \sin\theta \sin\phi$). We can map each location x on the ground to frequency using

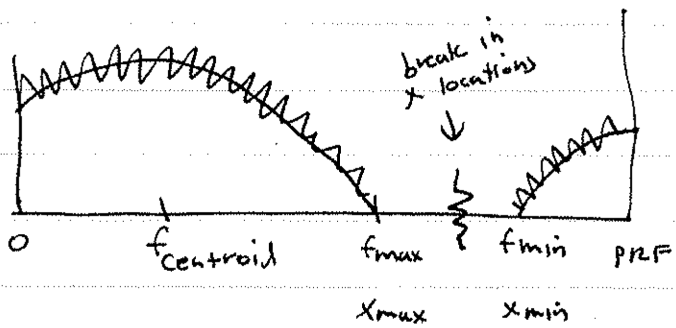
$$f_D = \frac{2v}{\lambda} \sin(sq) = \frac{2v}{\lambda} \frac{x}{r}$$

here an object at $f_D = f_{min}$ is at x_{min}, etc.

Note also that the return is weighted by the antenna pattern, somewhat distorting the image.



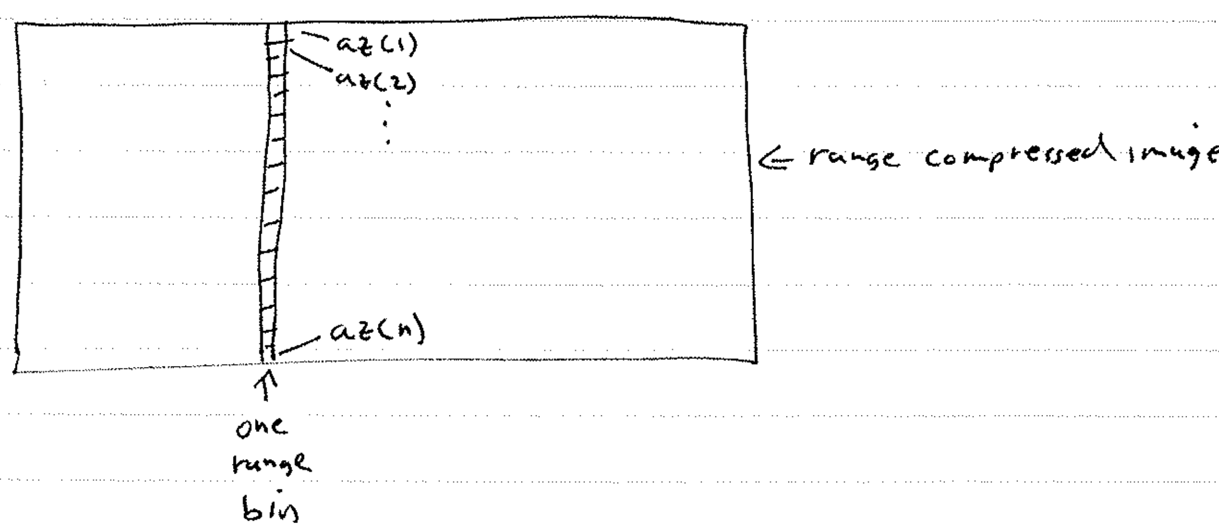
If we were to evaluate the above image (form the image) using an fft, though, the plot would look like



The image here is in two pieces which must be put together to form the full patch without any discontinuities.

One way around this is to "steer" the image to zero Doppler, that is force the antenna beam to peak at zero frequency as if the boresight were truly at zero. ~~The~~ Then, if we relate x positions to the antenna boresight, we can more easily visualize the mapping onto the surface.

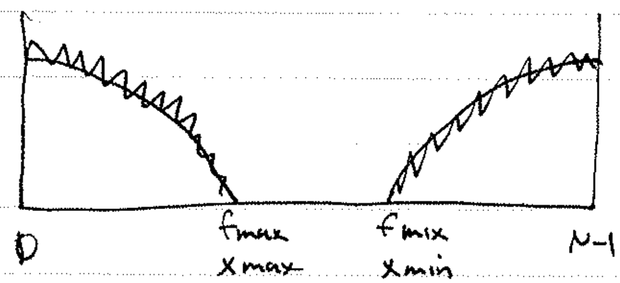
We "steer" the beam by multiplying the azimuth signal by a complex carrier equal ~~to the range~~ in frequency to the negative of the Doppler centroid. If az(j) represents a sequence of azimuth values at a constant range, such as



we simply form the product

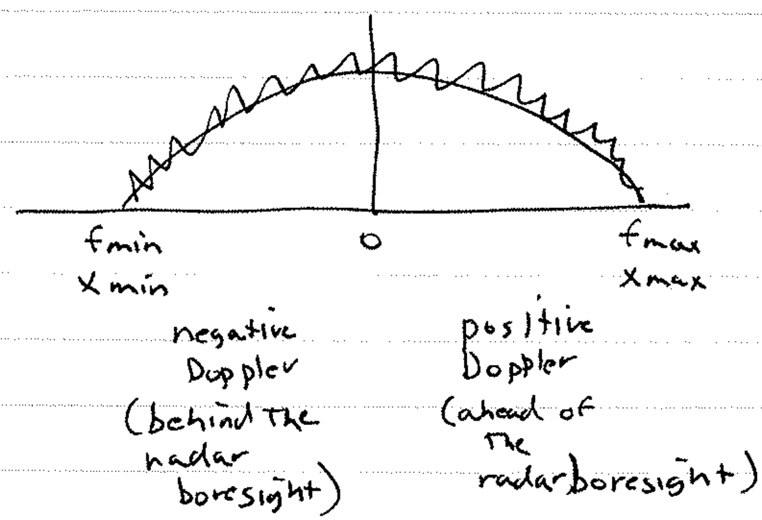
$$az'(i) = az(i) \cdot e^{-j\, 2\pi\, f_{centroid} \cdot i/prf}$$

before transforming. The resulting unfocused image then would be the shifted version



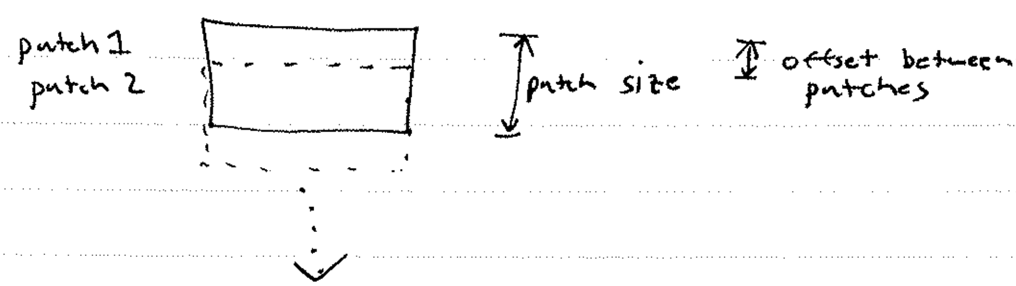
fft bin space

or, plotted with zero in the middle

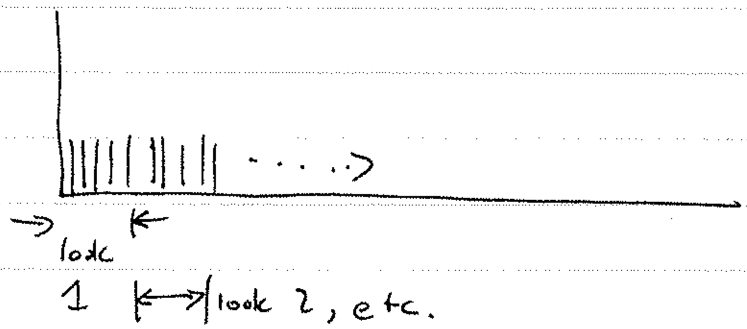


A multilook unfocused processor

Let's design a multilook version of the unfocused SAR processor. What we'll need is a method to process patch by patch and overlay the looks to form an image:



Also, let's use all the available data:



We'll assume we have data from ers-1 with

$r_0 = 850$ km

prf = 1679 Hz

$v = 75\ 0$ m/s

$\lambda = 0.0566$ cm

Also assume we have successfully range compressed the data, or at least can do so easily.

We'll yet assume that the swath is narrow enough that we can treat the entire width as if it were at range r_0.

First, how many pulses should ~~be~~ we ~~average in~~ transform in each look? Start with our resolution:

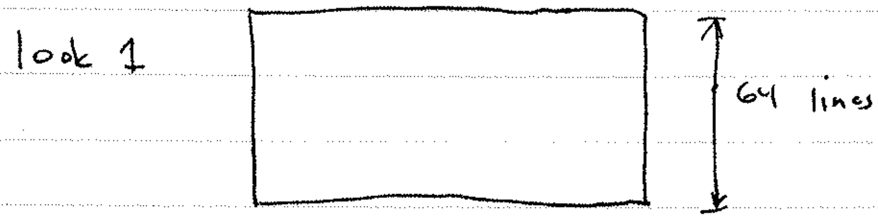
$$\delta_{az} = \sqrt{\lambda r_0} = 219 \text{ m}$$

The pulse spacing in azimuth is $\frac{75\ 0 \text{ m/s}}{1679 \text{ Hz}} = 4.497$ m, so the minimum burst size is

$$\frac{219 \text{ m}}{4.497 \text{ m}} = 48.7 \text{ pulses.}$$

Rounding up to the next power of 2 gives 64 pulses

So our patch size is 64 pulses:



Next we must evaluate the centroid. Say it occurs at $f_{centroid}$. We then can implement the single look unfocused processor by

1) range compressing 64 lines
2) multiply each value by its azimuth variation due to the boresight not being at zero:

$$az'(i) = az(i)\, e^{-j 2\pi f_{centroid} \cdot i/1679}$$

This is duplicated for each range bin

3) Calculate the Fourier transform in azimuth.
4) "Unpack" transform so zero is in the middle.

Now, we can take the next 64 lines and create a second "look". But the radar has moved forward by 64 pulses, or about 290 meters. But to calculate the offset, we need to relate the offset in meters by the radar to ground distance in our image.

Remember that "bins" in azimuth are formed in the unfocused processor through an fft operation. How much frequency is contained in each bin?

1) Transform length: 64 bins
2) Radar prf: 1679 Hz

Hence the frequency resolution $\delta f = \frac{1679}{64} = \underline{26.2}$ Hz.

How much change in x, then, per bin?

$$f = \frac{2v}{\lambda} \frac{x}{r}$$

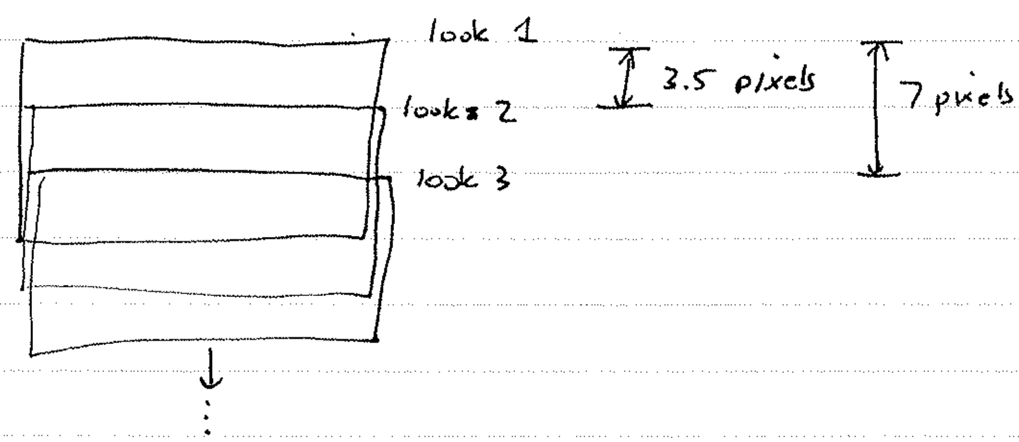
$$26.2 = \frac{2 \cdot 7550}{0.0566} \cdot \frac{x}{850000}$$

$$\Rightarrow x = \underline{\underline{83.476}} \text{ m.}$$

Our total shift, remember, was 290 meters. In pixels it is just

$$\frac{290}{83.476} = 3.47 \text{ pixels}$$

Hence our algorithm is



We can't easily shift half a pixel, but we can just let the total shift for look n to be ~~(n-1) 3.5~~ nint ((n-1) * 3.5) where nint( ) is the nearest integer.

This results in a multilooked, unfocused, beam-steered processor.