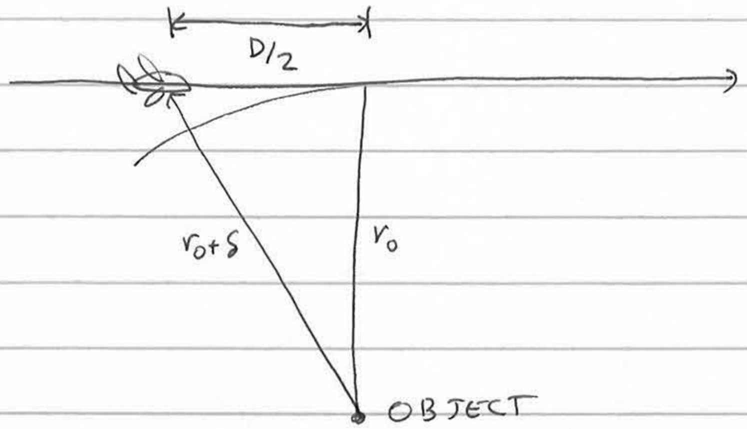
Derivation of Unfocused Processor Resolution

In class we stated without proof that $\delta_{at,\,unfocused} = \sqrt{r\lambda}$. There a couple of ways this can be derived. The most common argument is the following:

Radar flying by an object:



The unfocused algorithm is equivalent to summing coherently the reflected waves from the object, and the resolution depends on how much relative motion can be tolerated before the waves cease to add <u>approximately</u> in phase. Call this distance D, and if we center it at the object we have a relative motion of $\pm D/2$. [This is in fact the exact algorithm for the zero frequency bin in our unfocused Fourier transform processor]

Hence

$$(r_0+\delta)^2 = r_0^2 + \left(\frac{D}{2}\right)^2$$

$$r_0 + 2r_0\delta \approx r_0^2 + \frac{D^2}{4}$$

$$2r_0\delta \approx \frac{D^2}{4}$$

$$D \approx \sqrt{8r_0\delta}$$

Setting $\delta = \frac{\lambda}{8}$ for our condition of adding in phase, which corresponds to a two way distance difference of $\frac{\lambda}{4}$ or $90°$, we obtain the result shown in class

$$D = \sqrt{r_0 \lambda}$$

Other criteria, say $\frac{\lambda}{16}$, for $45°$ tolerance, yield for example

$$D = \sqrt{\frac{r_0 \lambda}{2}}$$

Note that velocity does not enter into the argument, only the total phase accumulated.

Another derivation

Another derivation follows from an uncertainty-principle like argument. We are try to determine the location of a point on the ground by measuring its doppler frequency but from a moving platform.

We thus have an uncertainty based on platform motion during the Δt observation interval, and an additional uncertainty due to our ability to discriminate frequencies by integrating over the same interval.

Each of these contributes to "smearing" of the impulse response. We can model the effective total resolution then by the convolution

$$\text{rect}\left(\begin{matrix}\text{platform motion}\\ \text{smearing}\end{matrix}\right) * \text{rect}\left(\begin{matrix}\text{frequency uncertainty}\\ \text{smearing}\end{matrix}\right)$$

Platform uncertainty: $\Delta x = v \Delta t$

Frequency uncertainty: $\Delta f = \frac{1}{\Delta t}$

Now, we relate frequency to position through

$$f = \frac{2v}{\lambda} \frac{x}{r}$$

hence

$$\Delta f = \frac{2v}{\lambda} \frac{\Delta x}{r}$$

So our impulse response will be, according to our simple model,

$$\underbrace{\mathrm{rect}\left(\frac{x}{v\Delta t}\right)}_{\Delta x \text{ from motion}} * \underbrace{\mathrm{rect}\left(\frac{x}{\frac{\Delta f \lambda r}{2v}}\right)}_{\Delta x \text{ from frequency unc.}}$$

The length of this impulse is the sum of the widths:

$$\text{length} = v\Delta t + \frac{\Delta f \cdot \lambda r}{2v}$$

$$= v\Delta t + \frac{\lambda r}{2v\Delta t}$$

To minimize this, differentiate and set equal to zero:

$$\frac{d(\text{length})}{d\Delta t} = v - \frac{\lambda r}{2v} \frac{1}{\Delta t^2} = 0$$

$$(v\Delta t)^2 = \frac{\lambda r}{2}$$

$$\Delta x = \sqrt{\frac{\lambda r}{2}}$$ ← our approx. result from before.

Once again velocity drops out of the equation.

Discussion question: Do we need classical Doppler to obtain images, or can point, shoot, and jump ~~[illegible]~~ approaches work?