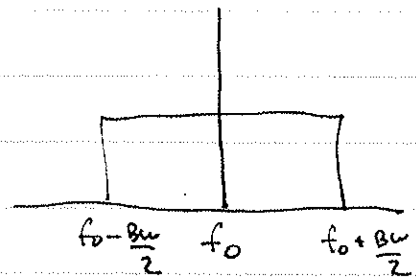
GP265/EE355 

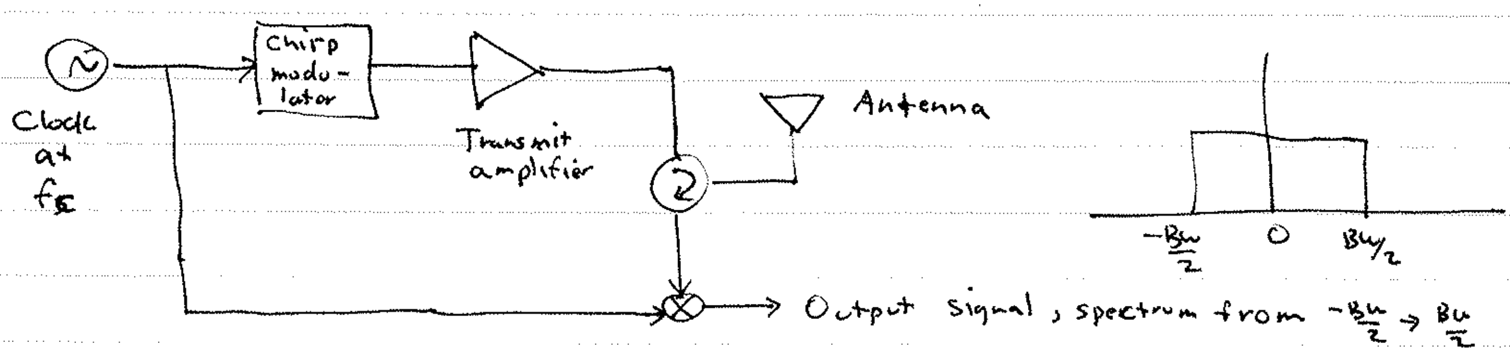
<u>One more range processing topic – I/Q vs offset video</u>

Before we leave the subject of range processing, we need to discuss the various ways the range data are recorded and sent to the processor. The two main formats used are I/Q, for in-phase and quadrature, where complex signals are stored, and offset video, where only real samples are saved.

Suppose we transmit a chirp of bandwidth BW around a carrier frequency $f_0$. The output spectrum then looks like

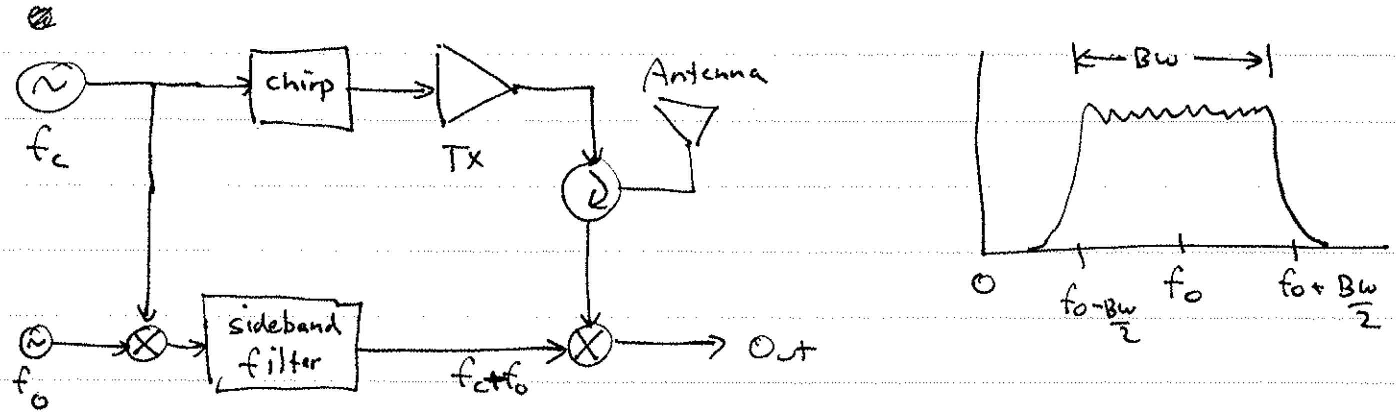



Suppose further that our radar design is the following:




Our signal to the digitizer varies from $-BW/2$ to $BW/2$ in frequency, and is sampled by a complex (or in-phase/quadrature) A/D converter. However, often it is difficult to design an accurate I/Q A/D converter, so sometimes a different scheme is used.

Our received spectrum varies in frequency from $f_c - BW/2$ to ~~BW/2~~ $f_c + BW/2$. Instead of mixing it with a clock at $f_c$, we can mix it with a slightly offset frequency $f_c + f_o$ such that $f_o > BW/2$. Now look at our system and output spectrum:



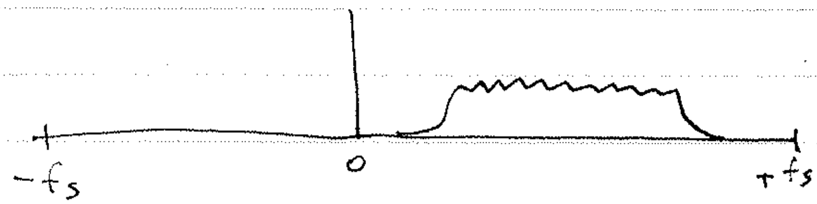

Now our output spectrum contains only positive frequencies, that is, the chirp never goes negative in frequency space. This has one desirable consequence: we can reconstuct the chirp signal from only its real part. We can use a real-valued quantizer rather than the complex quantizer we needed before.

However, since we will be using real samples only, we must sample the real part of the waveform at twice the bandwidth in order to obtain the full analytic signal. Hence the total number of data points in our echo remains constant, we need to sample only the real points but must sample twice as oftem.

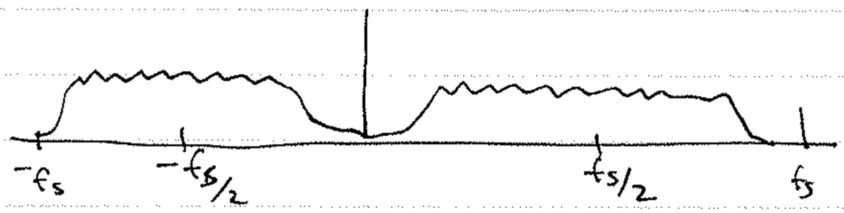
Note the added complexity of the radar when we use the offset-video approach. We have a tradeoff between the use of a complex quantization scheme and the use of a more complicated receiver. Which is easiest for any use is an implementation choice that could depend on many items.

## Spectrum of offset video signal

When we digitize the offset video signal, what do we retain in terms of the original information in the signal? Suppose our echo ~~~~ $s(t)$ is offset as below, and sampled at $2f_s$

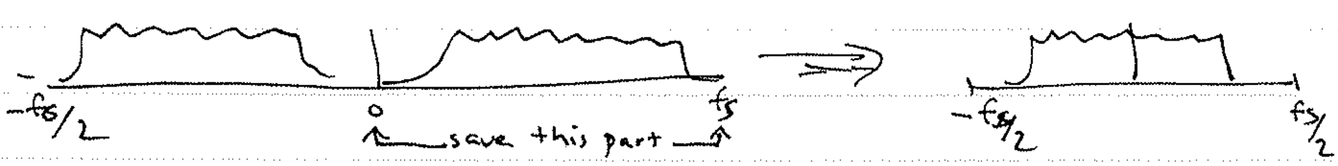



Now consider the real part of $s(t)$ only. It will have a double-sided spectrum as follows, on the same scale:




where for each positive frequency coefficient there is a corresponding negative frequency coefficient such that $S(-f) = S^*(f)$. When the bandwidth of the signal fits nonetheless between 0 and $f_s$, either of these sidebands contains sufficient information to reconstruct the desired echo. The information is duplicated by the $S(-f) = S^*(f)$ relation, and we need save only one side in our processing.

Thus, we can simply retain half of our signal in the spectral domain. Recalling that we oversampled by 2 initially, we are also back to a non-oversampled situation. ~~One ca~~ Since our signal appeared on an <u>effective</u> carrier of $f_s/2$, we can shift the spectrum by that amount to remove its effects at the same time:

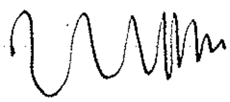


## Processing I/Q vs Offset video data

Processing offset video data differs from the i/q processing we have used so far in that we must convert the real-sampled signal we digitize to a complex signal for processing, generating the ~~and~~ analytic signal.

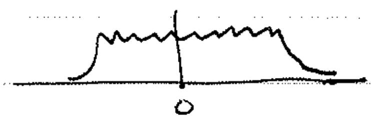
First, let's graphically review the i/q processing.

① create reference



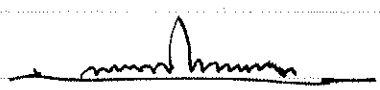
② transform reference

③ transform signal



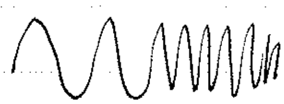
④ multiply $S(f) \cdot R^*(f)$

⑤ Inverse transform

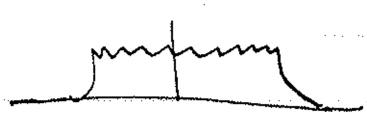


Now, let's look at more detail for offset video processing

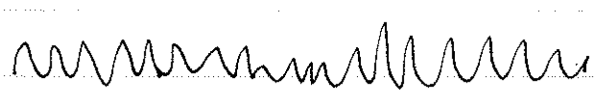
① create reference



② transform reference



③ copy signal into an array of double-length because of 2x sampling

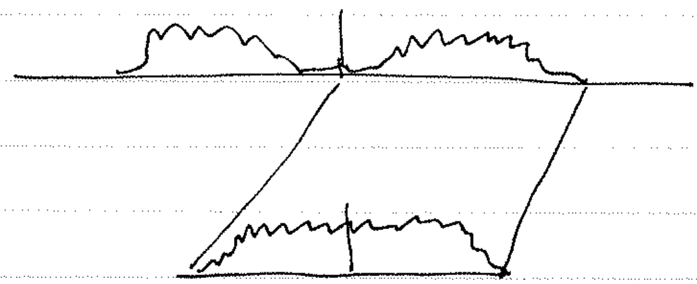


← real part only of complex array

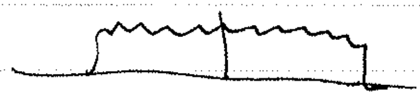
(4) transform signal 

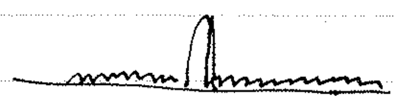
Note spectrum is two-sided because it is real only.

(5) save one "sideband" and copy into new, single length, transform array



(6) ~~inverse transform~~ multiply this shifted $S(f)$ by $R^*(f)$



(7) inverse transform 

Thus two slightly different algorithms are required for these two basic situations. From here on out we will assume all data are in complex sampled format, but you should be aware that offset video systems exist, have some implementation advantages, and need to be processed differently than i/q radars.

<This material is found in Chap. 4. Read it for an overview of image formation>

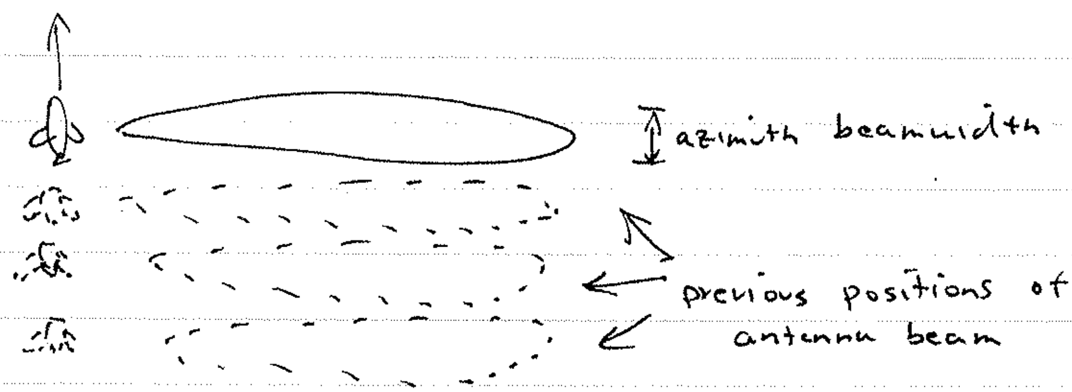
## Azimuth resolution and image formation

Now we are ready to proceed to azimuth image formation, that is how we generate resolution in the other dimension. Lets begin by looking at a real-aperture system, as opposed to a synthetic aperture system.

### Real Aperture Radar

In this case we have a platform flying along at constant velocity imaging a swath. Assume we have obtained fine range resolution via proper range modulation and processing.

Top view




If we were to pulse the radar occasionally, we could "stack" each range line, eventually forming an image of the surface. We need pulse the system only once each time the plane flies the width of the ground projection of the antenna beam, although we could pulse it much faster.

We know how to calculate SNR for this radar already. How does it perform in terms of resolution?

we cannot distinguish scatterers at different along track positions within the antenna beam.

Our azimuth resolution in a RAR is just the ground projection of the antenna:

$$\delta_{az\,(RAR)} = \frac{r\lambda}{l}$$

as before. Let's look at a few cases: (typical values shown)

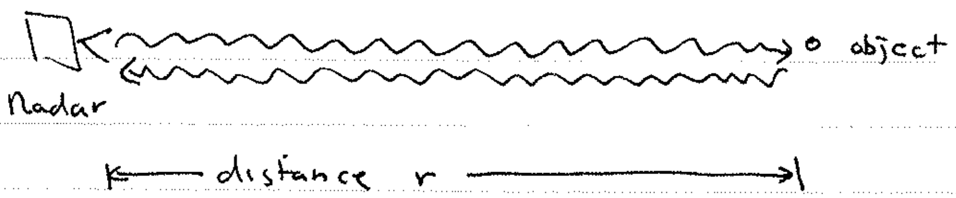
| | $\lambda$ | $r$ | $l$ | $\delta_{az}$ |
|---|---|---|---|---|
| Airborne L-band radar | 0.24 | 15 km | 1 | 3,600 |
| Airborne X-band radar | 0.03 | 15 km | 1 | 450 |
| Spaceborne L-band radar | 0.24 | 800 km | 10 | 19,200 |

While the X-band resolution is only $4\frac{1}{2}$ Stanford stadiums in extent, even that one is too large for most uses. Obviously the spaceborne systems are much too coarse for almost any practical situation. We need, therefore, to be more clever at exploiting the characteristics of the radar echo to generate finer resolution. (Recall that in range we have looked at meter-scale resolutions)

In order to do this, let's consider some of the properties of a radar echo.

## Phase of a radar echo

Consider the relation between distance and phase in radar echoes:




Every time an EM wave propagates its wavelength its phase advances by $2\pi$ (a definition). Thus, we can relate range $r$ to phase $\phi$ using the following

$$\phi = \frac{4\pi}{\lambda} r$$

If the range from an object to the radar is a variable function of time, $r(t)$, so is the phase

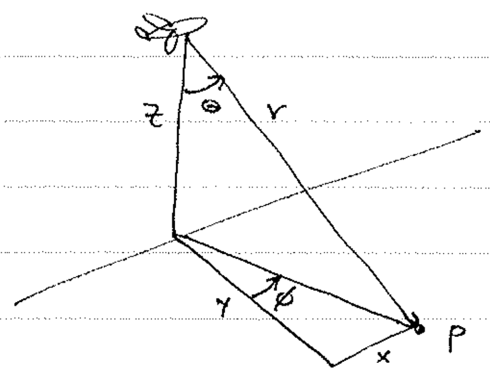
$$\phi(t) = \frac{4\pi}{\lambda} r(t)$$

Strictly speaking, if the radar signal is not monochromatic then the phase will be less well defined. But let's assume that the modulation (from the chirp) we put on the radar is small compared to its operating frequency, usually a good assumption. Then we can treat the signal as if all energy were concentrated at a single frequency.

For very wide-band or low-carrier-frequency radars, clearly this assumption is invalid. In that case what we do is to reformulate the problem in terms of ~~side~~ time delays rather than phases. It is a parallel development to what we are doing here, and we won't repeat it as it is a specialized application.

Refresher on imaging geometry

Here is our generalized imaging geometry:

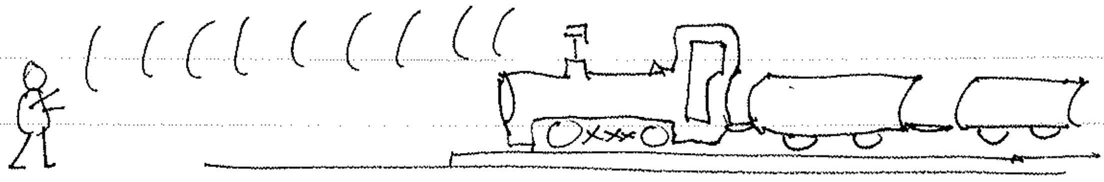



We'll need this in our investigation of the echo from various points in our illuminated beam.

We are going to separate our echoes along-track using the Doppler effect. Calculating the along-track Doppler component is the first step toward achieving higher azimuth resolution.

The Doppler effect

You have probably seen the Doppler effect and how it relates to sound echoes. This illustration probably looked something like this:




Because of the train's motion toward you, each wavefront is emitted a little closer to the previous waveform than we would expect from the wavelength. In fact the new wavelength $\lambda'$ is shorter by the amount the train travels in the time required for one wavelength, or

$$\lambda' = \lambda - v \cdot \Delta t$$

$$= \lambda - \frac{v\lambda}{c}$$

where c here is the speed of sound rather than light. Then letting the speed of the train toward the listener be v:

$$\lambda' - \lambda = \frac{-\lambda v}{c}$$

or

$$\frac{\Delta\lambda}{\lambda} = -\frac{v}{c}$$

In the case of frequency $f' = \frac{c}{\lambda}$, ~~who~~ we can restate the above as

$$\frac{c}{f'} = \frac{c}{f} - \frac{v}{f}$$

Hence

$$\frac{f'}{c} = \frac{f}{c-v}$$

$$\frac{f'}{f} = \frac{c}{c-v}$$

$$\frac{f'-f}{f} = \frac{c}{c-v} - 1$$

$$\frac{\Delta f}{f} = \frac{v}{c-v} \approx \frac{v}{c}$$ ← assume $v \ll c$, the non-relativistic Doppler relationship

$$\Delta f = \frac{v}{\lambda}$$

In the radar case, because we have two-way travel, a factor of 2 is involved. Also we can generalize the relation for an arbitrary velocity vector v for the target to the radar and a unit vector from the radar to the object u, and obtain

$$\boxed{\Delta f = \frac{2\, v \cdot u}{\lambda}}$$

where $v \cdot u$ represents the component of velocity toward the radar.