# Other range processing issues

## Shape of the impulse response

Consider the equation we developed previously linking the processed image to actual reflectivity through the autocorrelation function of the transmitted waveform:

$$i(t) = b(t) * \left(r(t) \star r(t)\right)$$

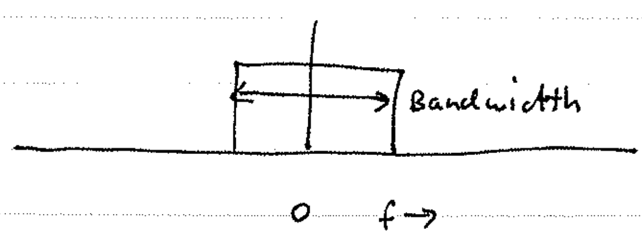
Hence we can describe the range response of the radar as

$$\text{range impulse response} = r(t) \star r(t)$$

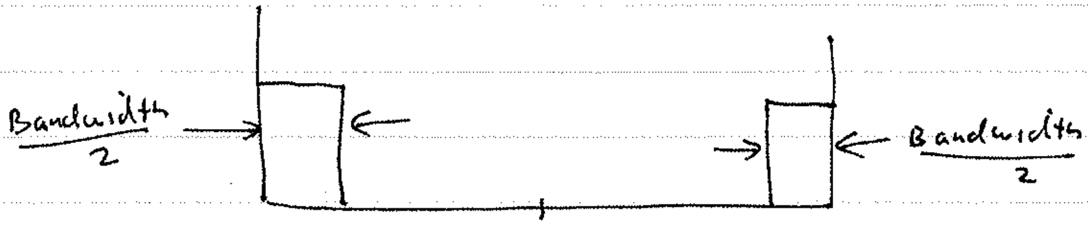
or, in the frequency domain as

$$\text{Spectrum of range impulse response} = R^*(s)\, R(s) = |R(s)|^2$$

For a chirp signal with high time-bandwidth product, and centered at zero frequency, we have the following approximate spectrum:




→ As an aside, let's remind ourselves how this looks in the discrete domain of the FFT algorithm:




0 $\longleftarrow$ bin # $\longrightarrow$ N−1

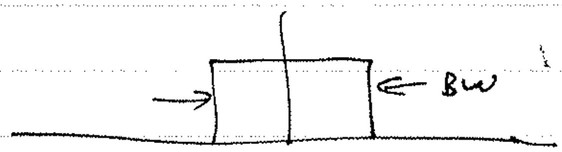
0 $\longleftarrow$ Frequency $\longrightarrow$ $f_s$

or

0 $\qquad\qquad \frac{f_s}{2} \qquad\qquad$ 0

+ frequency $\qquad\qquad$ − frequency

Back to the continuous world plot:

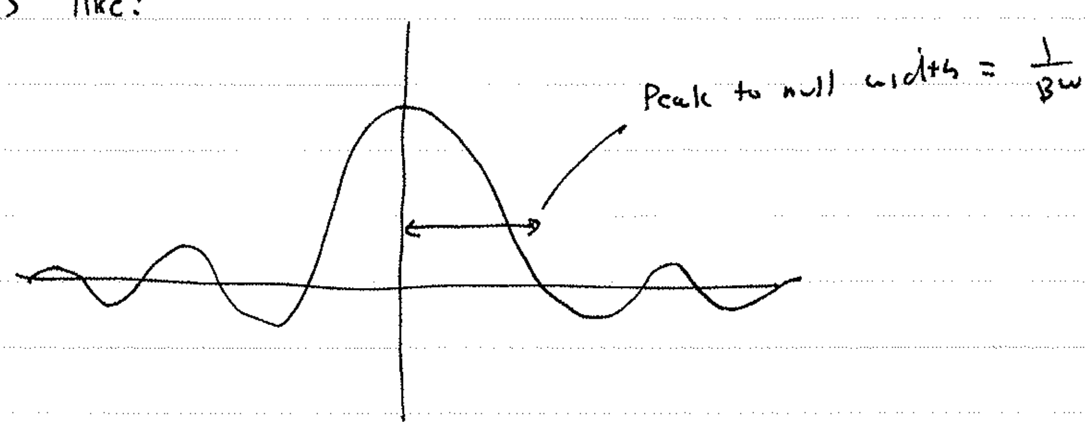



We know that the impulse response is the transform of this function:

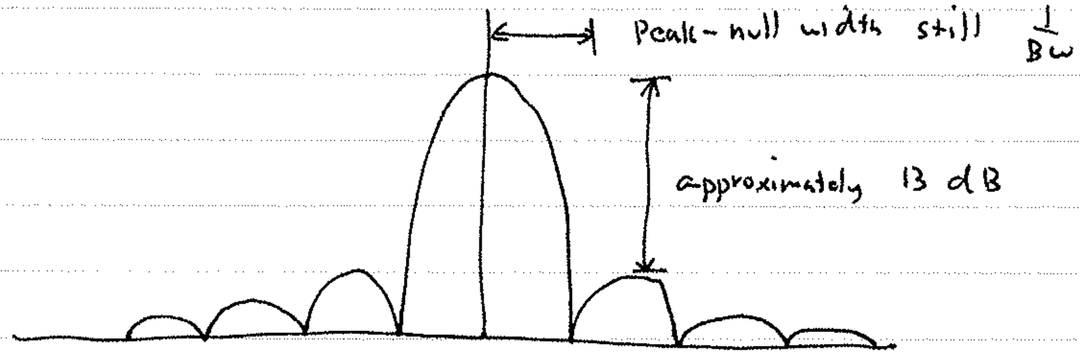
$$\text{impulse response} = \operatorname{sinc}(BW \cdot t)$$

$$= \frac{\sin(\pi \cdot BW \cdot t)}{\pi \cdot BW \cdot t}$$

which looks like:



Plotted on a dB scale:




We can calculate the height of the first sidelobe for an ideal sinc function:

Peak value = sinc (0)
1st sidelobe = sinc $\left(\frac{3}{2}\right)$

$$\Rightarrow \text{Amplitude ratio} = \frac{|\text{sinc}(0)|}{\left|\text{sinc}\left(\frac{3}{2}\right)\right|} = \frac{1}{\left|\frac{\sin\left(\pi\cdot\frac{3}{2}\right)}{\frac{\pi\cdot 3}{2}}\right|} = \frac{3\pi}{2} = 4.712$$

In dB, ratio $= -20 \log 4.712 = -13.465$ dB (note negative sign)

The second sidelobe is at $-17.9$ dB, and so forth.

The effective width of the sinc we saw was $\frac{1}{BW}$. How much did we "compress" the original pulse?

The pulse initially had a pulse length of $\tau$. Hence the ratio of improvement is

$$\tau/(1/BW) = \tau \cdot BW$$

or, the time-bandwidth product.

In terms of the original chirp slope, s,:

$$BW = s \cdot \tau$$

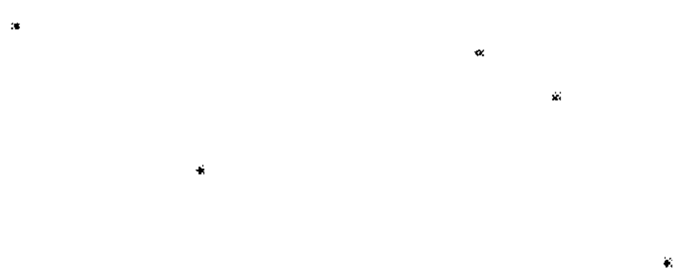
$$\text{hence } \tau \cdot BW = s \cdot \tau^2$$

So the chirp improves compression by a factor of the time bandwidth product, and generates sidelobes with a peak starting at -13 dB and getting progressively smaller.

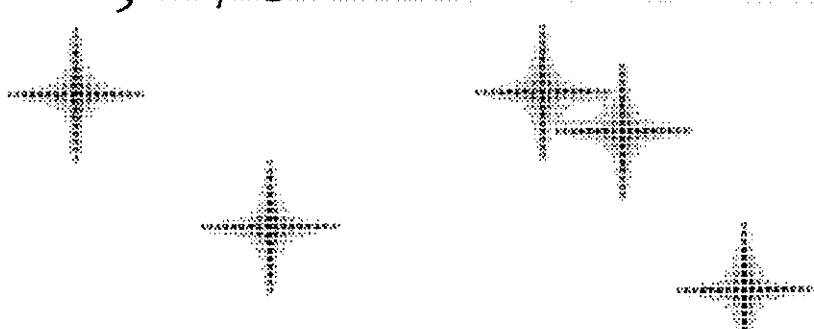
What does the presence of sidelobes in a radar signal imply? Any target will not only generate its main lobe response but also a "trail" of sidelobes. In fact we could have anticipated this because we knew that the image was the convolution of the true brightness distribution with the impulse response, in our case a sinc function.

Here is a sample image, consisting of a few bright point reflectors, and sidelobes:

~~Also~~ Original distribution:



Radar image w/ sidelobes:



Note here that we have used a two-dimensional sidelobe distribution pattern, that is, we have included sidelobes in both range and azimuth in a simulation of the full imaging situation.

<u>Sidelobe ~~reduction~~ reduction</u>

Suppose we have a situation where the sidelobes are troublesome, such as in a landing radar at SFO. Here the need is obvious, but for images it is also clear that we will be able to more easily infer the surface properties if the sidelobes are smaller.

We can calculate the amount of energy in the main lobe of the sinc function compared to the amount in the sidelobes. We call this quantity the <u>integrated sidelobe level</u>, or ISLR, of the system (acronym stands for integrated sidelobe ratio). In one dimension, for our sinc we have

$$ISLR = \frac{\int_1^\infty \operatorname{sinc}^2(x)\,dx}{\int_0^1 \operatorname{sinc}^2(x)\,dx}$$

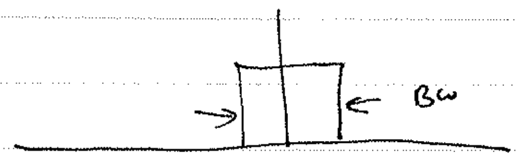
$$\approx 0.106 \quad , \text{or} \quad -9.75 \text{ dB}$$

Fully 10% of the energy is in the sidelobes. In 2-D, over 20% of the energy is in the sidelobes, and the ISLR is only − 6.9 dB.

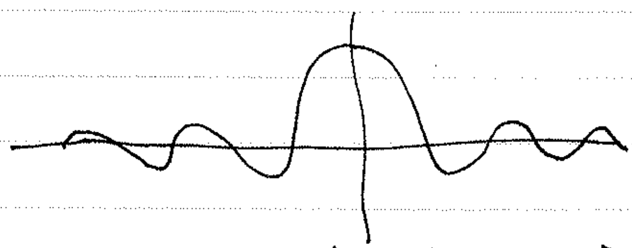
How can we improve both the peak sidelobe level of −13 dB and the ISLR of −9.7 or − 6.9 dB? The answer is by <u>weighting</u> the spectral response of the radar.

Weighting

Weighting refers to changing the frequency response of the system. Consider the spectrum we have been investigating:



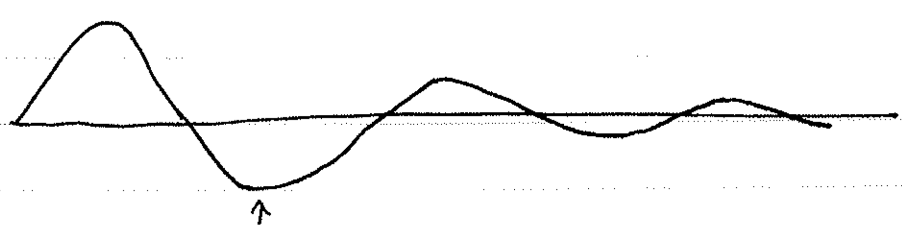

We can describe this function as rect $\left(\frac{f}{BW}\right)$, a rectangle function, centered at 0, of width BW. Remember that it transforms into a sinc function:



What are some properties of the sinc function?

1. Alternating sidelobes decreasing in amplitude
2. Evenly spaced sidelobes at unit spacing

How might we operate on this signal to reduce sidelobes? Consider a blowup of part of the sidelobe structure:



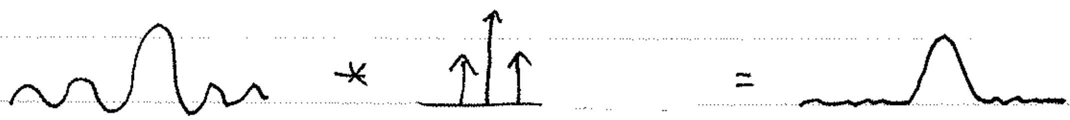
amplitude of this is about halfway between those of adjacent lobes, and negative

$\leftarrow \Big\{$ true for every sidelobe

So, amplitude sidelobe $n \approx -\frac{1}{2}$ (amp. of $n-1$ plus amp. of $n+1$)

Thus, if we were to add to every sidelobe the average of the values on either side, we would approximately cancel the sidelobes.

We can do this by convolving the impulse response with two $\delta$-functions spaced at 2 units; plus the original value:



Hence the simple signal processing operation of convolution with 2 $\delta$-functions, appropriately spaced, reduces the sidelobes!

But our implementation is even easier than this. Let's look at the same operation in the frequency domain:

Since the convolution with a pair of $\delta$ functions is equivalent to multiplying in the frequency domain by a cosine, simple weighting in the frequency domain also reduces sidelobes.

It is a straightforward exercise to show that the correct frequency of the cosine to be applied puts exactly one cycle on the spectrum:

We wish to replace each point with the sum of itself ($\delta(x)$) plus the sidelobe-cancelling pair ($\frac{1}{2}\delta(x-1) + \frac{1}{2}\delta(x+1)$):

Hence the total convolution is

$$\operatorname{sinc}(x) * \left[\tfrac{1}{2}\delta(x-1) + \tfrac{1}{2}\delta(x+1) + \delta(x)\right]$$

which transforms into

$$\operatorname{rect}(x) \cdot \left[\cos(2\pi x) + 1\right]$$

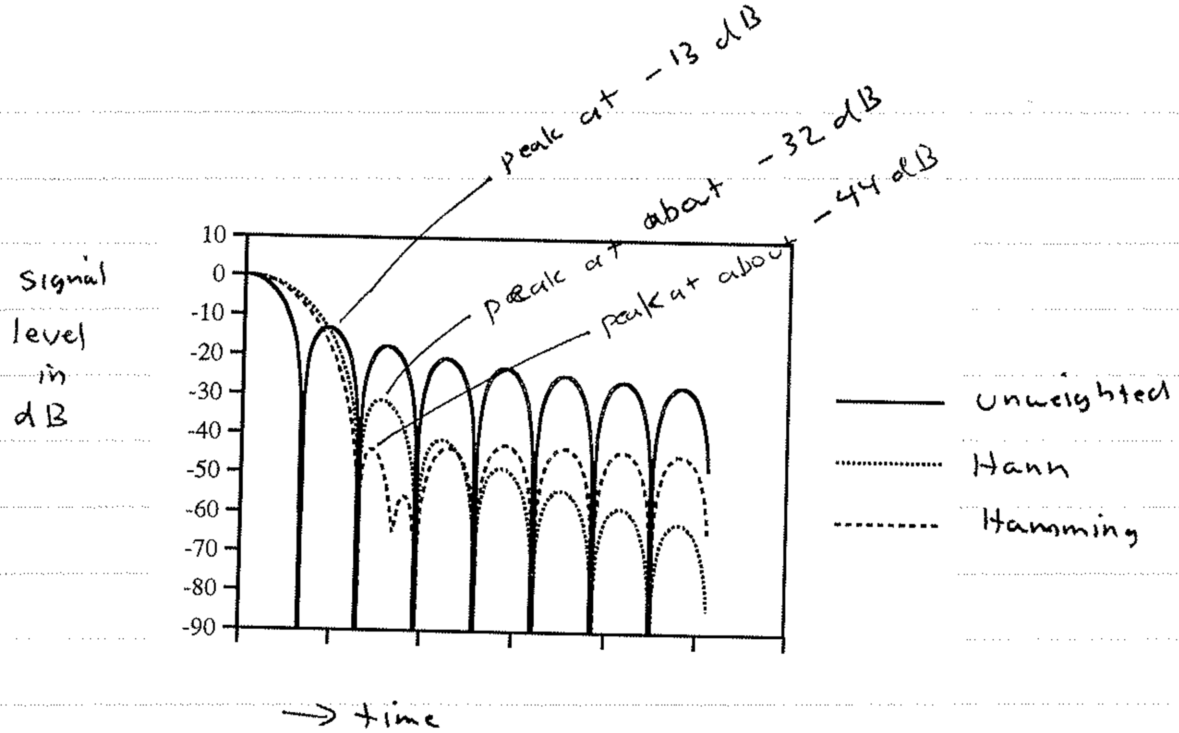
In fact there are many families of weighting functions that give various properties to the impulse response. The one we developed above is called the Hann function, or window, and is similar to the Hamming function. Several properties of each are given below:

Hann: $\dfrac{1 + \cos(2\pi x)}{2} = 0.5 + 0.5 \cos 2\pi x$

Hamming: $0.54 + 0.46 \cos 2\pi x$

Unweighted: $\underline{1}$

Plots of the widths and sidelobes are shown on the following page.




The ISLR's for these are <u>about</u>

Unweighted : -9.7 dB

Hann : -32 dB

Hamming : -34 dB

So we have achieved a large reduction in sidelobe peaks and ISLR at the expense of, in this case, a doubling of the resolution.

Discussion question: Why is the resolution about double?

<u>Summary</u>:

The shape of the impulse from a chirped radar is approximately a sinc-function.

Sidelobes can be manipulated through weighting.

We usually can trade off resolution vs. sidelobe energy.