EE355/GP265

Range modulation, processing, and pulse compression

[Read: Chapter 3 of the text "Matched Filter and Pulse Compression" for more on pulse compression.]

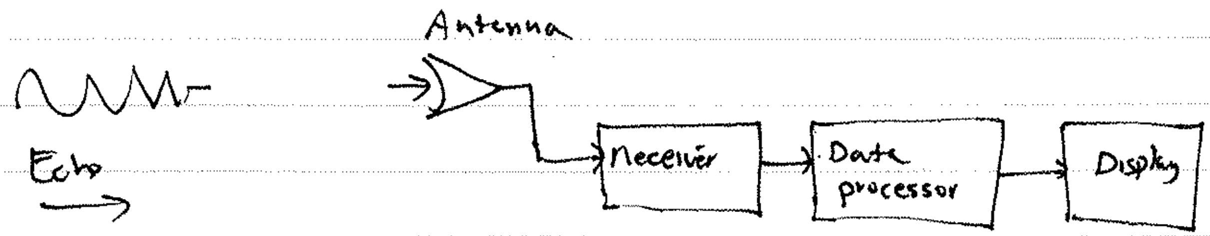
We have seen that the SNR of an imaging radar depends only on the energy of the transmit pulse, that is on the product of peak power and pulse length. Also, the resolution of the system depends on pulse length.

We thus have a design trade-off: To get very fine resolution we want to transmit very short pulses, but to obtain high SNR's we would have to use very high peak powers. Fortunately there is a way around this problem: range modulation (or coding) and pulse compression.

→ This is the first instance where we will see a substantial improvement of system performance following application of a signal processing algorithm. One of the main themes of this class is to examine how several signal processing techniques adapt the rather crude hardware of a radar system to become useful for detailed geophysical (or other) investigations.

The matched filter

Each point in a radar image reflects a version of the transmitted pulse back to the receiver. We would like to isolate this signal from surrounding noise and measure its amplitude accurately:



We can design a filter to optimally discriminate the known form of the radar echo from the background noise, which we'll model as a white Gaussian process. From our signal processing background, we know that an optimal filter in the least-squares sense is obtained if we use a matched filter, that is one with the conjugate response to the transmitted signal.

Suppose our radar transmits a waveform $s(t)$. The echo received at the antenna is a scaled version of that ~~puts~~ plus noise:

$$r(t) = \alpha \cdot s(t-\tau) + n(t)$$

where the argument $(t-\tau)$ reflects the time delay, α is a constant, and $n(t)$ is the noise. We want to find a filter $h(t)$ to maximize the SNR of the output of the system. In other words we want to maximize

$$\frac{\langle |h(t) * (\alpha s(t) + n(t))|^2 \rangle}{\langle |h(t) * n(t)|^2 \rangle}$$

where $h(t)$ is our filter and $\langle\,\rangle$ denotes expectation. Without explicitly deriving the solution here we eventually obtain

$$h(t) = s^*(-t)$$

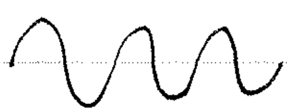
which is called the matched filter (see pp. 128-129 in text for derivation).

The transfer function of this filter is thus

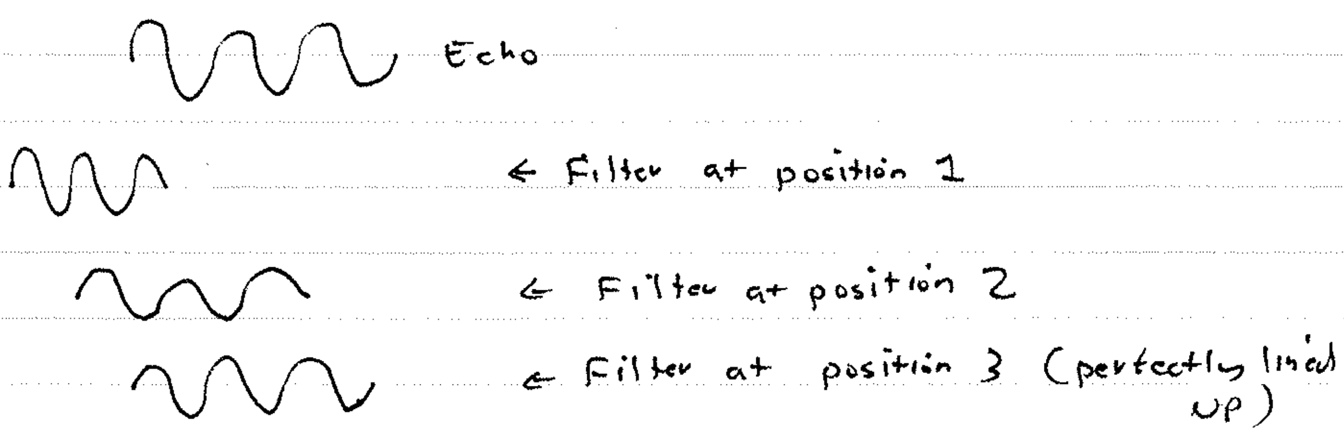
$$H(f) = S^*(f)$$

Hence by designing our receiver/processor to have the same spectral response, with inverted phase, as the transmit spectrum we can maximize SNR.

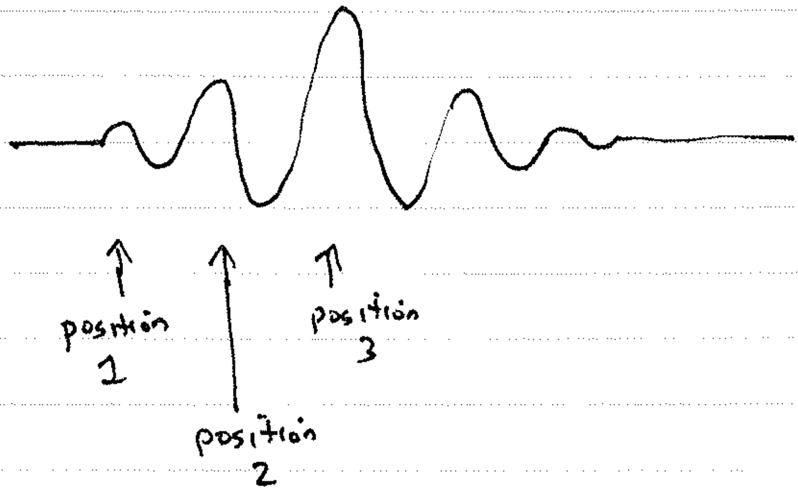
An illustration: say we transmit a short burst of energy corresponding to 3 cycles of signal:



Each echo will have these three cycles. What will be the response of a matched filter to this echo? The matched filter results from convolving the echo with its time-reversed version, or correlating with the transmitted signal. Recalling that we correlate by a shift, multiply, and adding process:



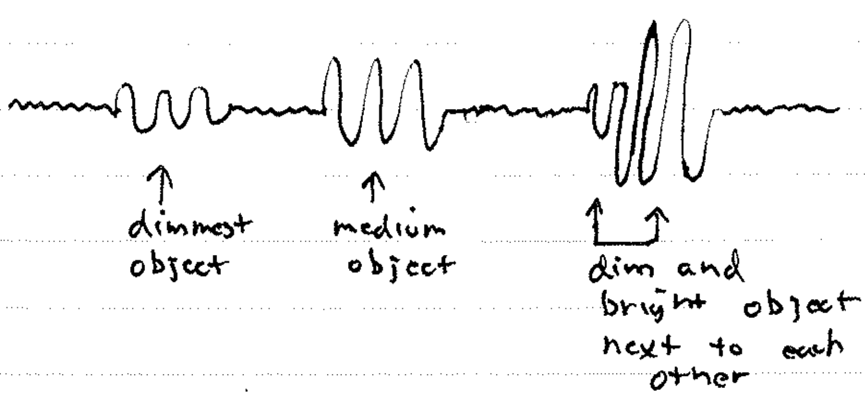
If we plot the output of the receiver as a function of filter delay, then, we obtain



At position 1, one cycle overlaps so there is a small response. At position 2, two cycles overlap. At position 3, the maximum, all three cycles overlap. This is the best estimate of location (the delay) and the highest SNR point of the output.

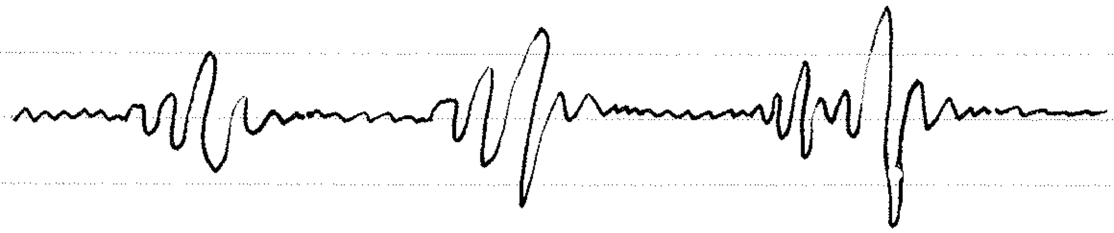
Let's examine the received signal and the "processed" signal, assuming a collection of a few bright points in an otherwise dim image.

Received signal:



Note the noise between the objects, and that the echoes from the two points at right merge.

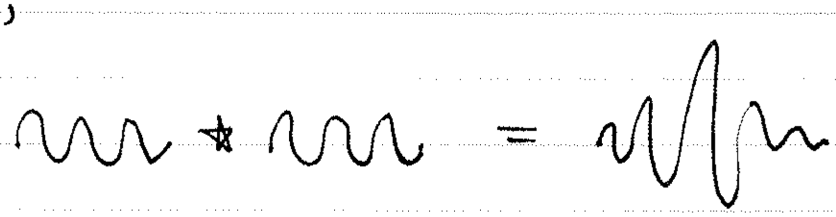
Processed signal:



In the processed case, the energy from each individual echo has been "compressed" toward the center of the pulse, resulting in a higher signal to noise ratio in the center and a better defined location. This plot represents optimally filtered data for the short pulse burst we transmitted.

<u>Range coding</u>

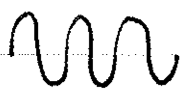
So we can obtain some increase in performance by optimally filtering our data by correlating the echo with a replica of the transmitted pulse. The <u>impulse response</u> of our system is then the correlation of our transmitted signal with itself, or its autocorrelation. In the case we examined,

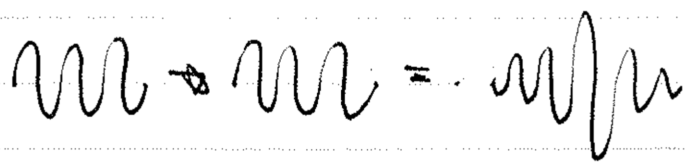
 ← impulse response

where we let ☆ denote correlation.

But we can do a lot better if we use again our signal processing knowledge to code the transmitted waveform cleverly. Let's code our burst by reversing the phase of each ~~sy~~cycle according to a predefined code, and simplify the result by sampling only at the points where ~~an~~ integral number of cycles overlap.

We'll begin with a code three cycles long, with all phases being "1". That is what we used before:

 is equivalent to 1 1 1 for three phases.

 is equiv. to $111 * 111 = 1\ 2\ 3\ 2$

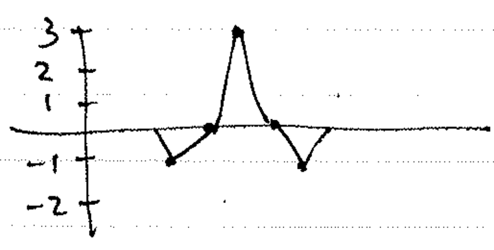
where 1 2 3 2 1 represent amplitudes of the processed return.
Now, substitute the code 1 1 -1 and correlate:

$$\begin{array}{ccccc} & & 1 & 1 & -1 \\ 1 & 1 & -1 & & \\ \hline 0 & 0 & -1 & 0 & 0 \end{array}$$ ← products sum to -1

$$\begin{array}{cccc} & 1 & 1 & -1 \\ 1 & 1 & -1 & \\ \hline 0 & 1 & -1 & 0 \end{array}$$ ← products sum to 0

$$\begin{array}{ccc} 1 & 1 & -1 \\ 1 & 1 & -1 \\ \hline 1 & 1 & 1 \end{array}$$ ← products sum to 3, etc.

Thus the output from this coded signal is



In this case the peak value is still 3, but non-peak values are all 0 or -1. Using the proper codes allows us to generate arbitrarily long sequences with this improved compression performance.

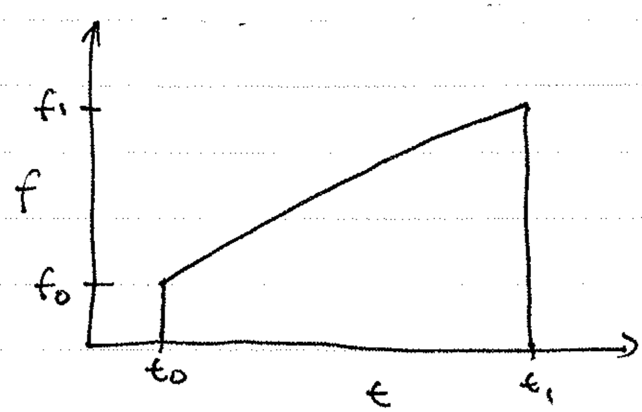
Analog codes

In fact few radars (imagers, anyway) use digital codes like this, because of a property known as "Doppler resistance". Since detection of the code involves estimating the phase of each cycle, if the echo is Doppler shifted the phase measurements become noisier.

We overcome this limitation by using an analog code that is less sensitive to such problems. We still can represent our impulse response by the autocorrelation of the transmit signal.

Chirps

The most widely used range modulation is the so-called chirp waveform, consisting of a linearly-swept frequency pulse. We can define it as in the following sketch:



← frequency increases linearly with time

Hence

$$f(t) = s \cdot t + f_0 \qquad t_0 < t < t_1$$

where s is the chirp slope, in Hz s^{-1}, and f_0 represents the starting frequency.

Sometimes a chirp can be defined in terms of its center frequency f_c:

$$f(t) = s \cdot t + f_c \qquad -\frac{\tau}{2} < t < \frac{\tau}{2}$$

where s is still the same slope, f_c is the center frequency, and the total pulse length is τ.

Phase is the integral of angular frequency, so the phase $\phi(t)$ for the chirp may be expressed as

$$\phi(t) = 2\pi s \frac{t^2}{2} + 2\pi f_c t + \phi_0$$

$$= \pi s t^2 + 2\pi f_c t + \phi_0$$

where ϕ_0 is an arbitrary constant of integration. Our transmitted waveform is hence

$$s(t) = \exp\left(-j\left[\pi s t^2 + 2\pi f_c t + \phi_0\right]\right)$$

Let's let $\phi_0 = 0$ for now to simplify matters.

Since the range of frequencies is from f_0 to f_1, the bandwidth of the chirp is $f_1 - f_0$. In terms of chirp slope s,

$$f_1 = f\left(\frac{\tau}{2}\right) = s \cdot \frac{\tau}{2} + f_c$$
$$f_0 = f\left(-\frac{\tau}{2}\right) = -s \cdot \frac{\tau}{2} + f_c$$

so the bandwidth $BW = s \cdot \tau$.

It's also useful to know the time-bandwidth product, $s \cdot \tau^2$.

Spectrum of chirp

Since the chirp frequency varies linearly with time, we would expect flat spectral density in its spectrum. In fact the chirp spectrum approaches this idealized limit if the time bandwidth product is large:

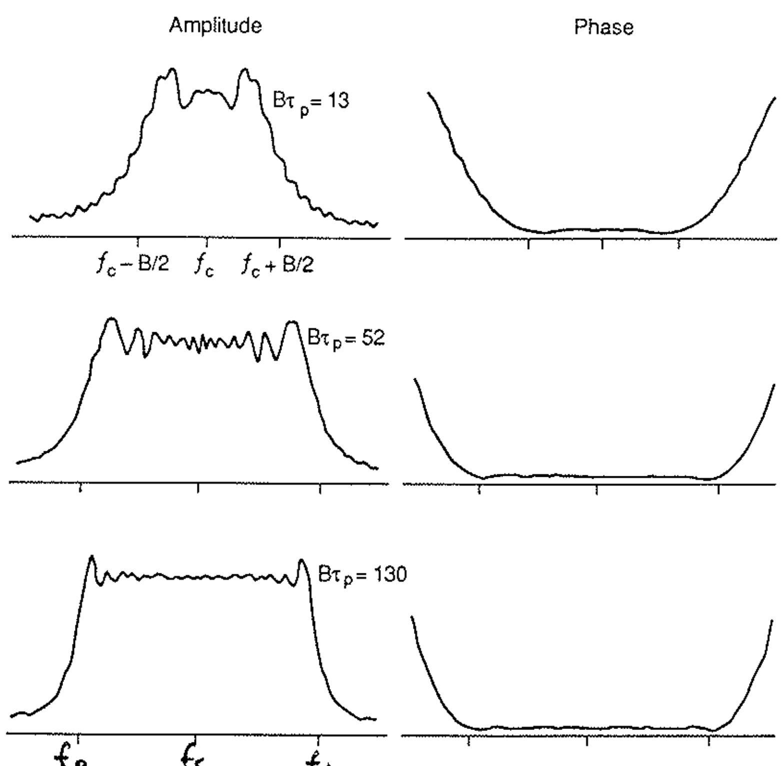


Figure 3.3 Amplitude and phase spectra of linear FM signals with various bandwidth time products. Phase shown is residual after removal of nominal quadratic phase (from Cook and Bernfeld, 1967 and after Cook, 1960. *Proc. IRE*, **48**, pp. 300–316. © IEEE)

Chirp Impulse Response

As before, we can estimate the impulse response of the system by examining the autocorrelation of the chirp waveform.

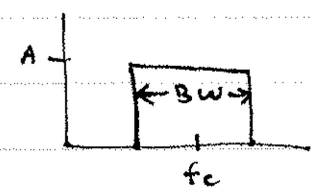
If $s(t) = \exp(-j[\pi s t^2 + 2\pi f_c t])$ for $-\frac{\tau}{2} < t < \frac{\tau}{2}$, the impulse response $i(t)$ is

$$i(t) = \int_{-\infty}^{\infty} s^*(t' - t)\, s(t' - \tau_{delay})\, dt'$$

But we can also evaluate the correlation in the frequency domain. For large time bandwidth products (> 100), we can describe the amplitude part of the chirp spectrum as

$$A \cdot \mathrm{rect}\left(\frac{f - f_c}{BW}\right)$$

or



The output is the product of this with its conjugate, canceling the quadratic phase term exactly, hence the impulse response is is the inverse transform of a rect of width BW, or a sinc function of width $\frac{1}{BW}$:

$$i(t) = \mathrm{sinc}(BW \cdot t)$$

offset properly in time and on the carrier frequency f_c. The compressed pulse width τ_{eff} is now $\frac{1}{BW}$, independent of actual pulse length τ. This will allow us to specify our resolution ($1/BW$) independently of our SNR (depends on τ).