EE355/GP265

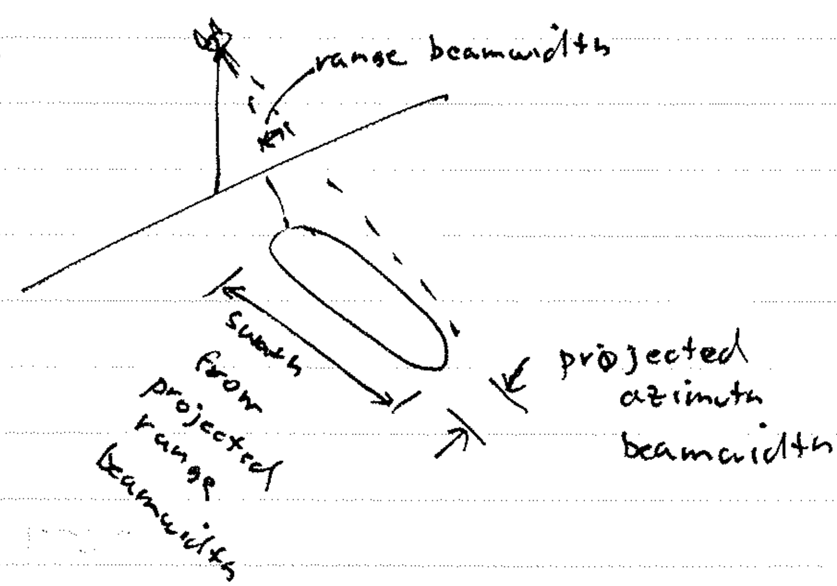
Pulsed Imaging Radar form of the Radar Equation

The form of the radar equation we have used so far does not account for pulsing the radar signal, that is, it is a steady state calculation assuming a continuous transmission. For many radars, and nearly all imaging radars, the transmission is broken up into pulses. This affects the radar equation.

Why? The received echo duration in time depends on the antenna pattern and imaging geometry, in addition to the transmit pulse length. As we will see, the echo can be described as the convolution of the actual backscatter distribution, weighted by the antenna pattern, ~~and~~ convolved with the transmitted signal.

Scattering area computation

Both a pulse-stretching viewpoint and a convolution viewpoint lead to a modification of the scattering area computation in the radar equation. Recall our calculation of illuminated area:



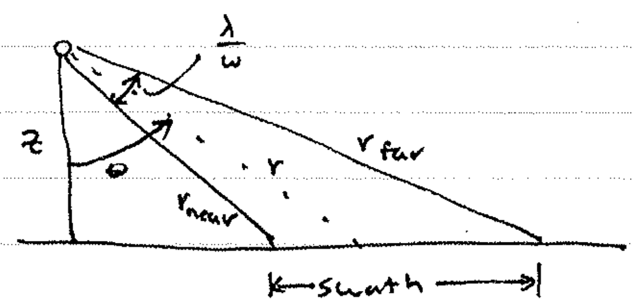
If the antenna size is $l \times w$ meters, the distance to the center of the swath r, and the look angle θ, we had

$$\text{projected range beamwidth (swath)} = \frac{r\lambda}{w\cos\theta}$$

$$\text{projected azimuth beamwidth} = \frac{r\lambda}{l}$$

$$\text{Area} = \frac{r^2\lambda^2}{lw\cos\theta}$$

Consider the range term in detail:



Suppose we transmit a signal that looks like a δ-function. Over what time will the echo be spread?

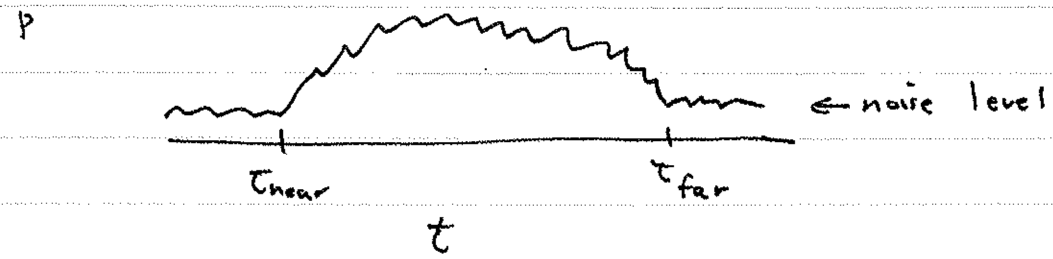
$$r_{near} = \frac{z}{\cos\left(\theta - \frac{\lambda}{2w}\right)}$$

$$r_{far} = \frac{z}{\cos\left(\theta + \frac{\lambda}{2w}\right)}$$

and since $\tau = \frac{2r}{c}$

$$\tau_{near} = \frac{2z}{c \cdot \cos\left(\theta - \frac{\lambda}{2w}\right)} \qquad \tau_{far} = \frac{2z}{c \cdot \cos\left(\theta + \frac{\lambda}{2w}\right)}$$

A graph of the received signal might look like:

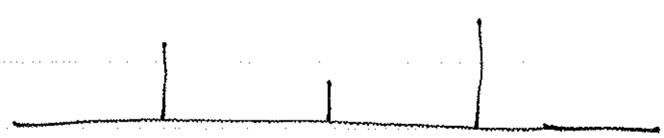


So the impulse transmit pulse is spread out over a range of times. If instead we were to transmit a pulse of finite duration, each point on the ground would return a duplicate of the transmit pulse, properly scaled in amplitude according to its reflectivity, propagation losses, and antenna weighting.

Hence if the image reflectivity and weightings could be expressed as a function of time $i(t)$, each ~~echo~~ point t_0 in the echo returns

$$i(t_0) \cdot T(t)$$

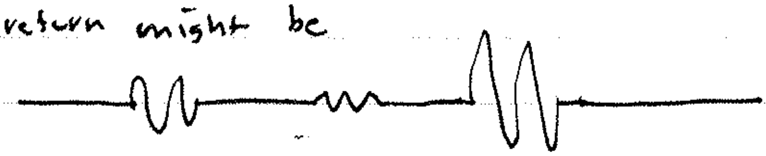
Say Reflectivity is a few δ-functions:



and the transmitted pulse is

$T(t) =$ 

then the return might be



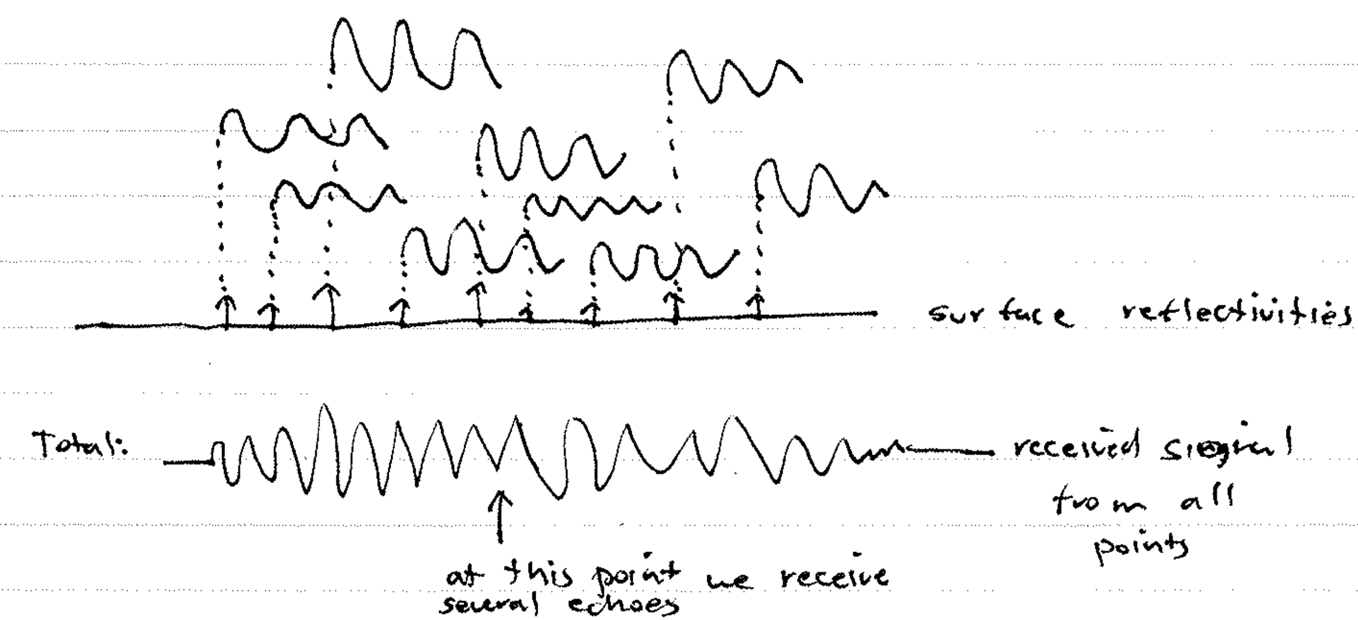
Here it is obvious that the echo $r(t)$ is the convolution of the image $i(t)$ with the transmit signal $T(t)$.

$$i(t) = r(t) * T(t)$$

This same relationship holds for a continuous distribution of $i(t)$. Hence the true reflectivity distribution is "smeared" out by the radar pulse -- this determines the radar <u>resolution</u>. We'll return to resolution in a bit, first let's finish the scattering area computation.

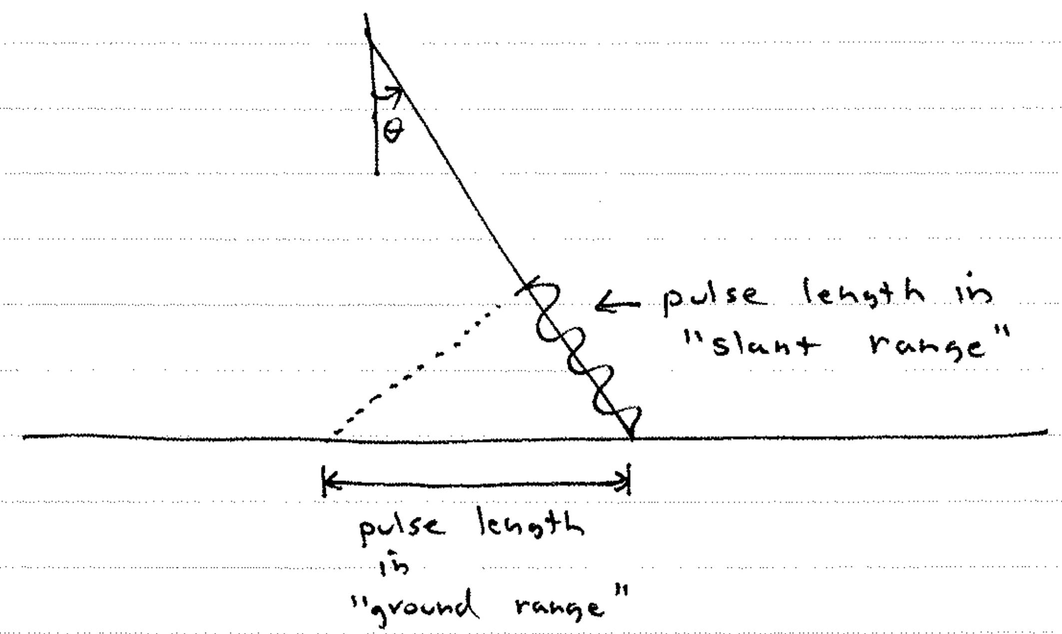
<u>Projected pulse in scattering area</u>

Because the true reflectivity distribution in time is convolved with the transmit pulse, at any given time in the echo we are receiving energy from more than one point on the ground. Here is a sample echo showing individual contributions from a number of discrete points on the surface:



In fact at any given point we receive contributions from all points within one physical pulse length. Hence the size of the scattering area in the range direction contributing to the echo at any given time is the length illuminated by one pulse length, projected onto the ground.

For the usual flat-earth geometry:



For a pulse length τ in time the "length" in meters is

$$\delta = \frac{c\tau}{2}$$

and on the ground

$$\delta_{gnd} = \frac{c\tau}{2\sin\theta}$$

Using this relation for the range extent of scattering area,

$$A_{scatt} = \underbrace{\frac{c\tau}{2\sin\Theta}}_{\text{range contribution}} \cdot \underbrace{\frac{r\lambda}{\ell}}_{\text{azimuth contribution}}$$

Substituting this relation for scattering area into our previous radar equation gives the pulsed version.

Peak vs. "average" power (!), or energy

Suppose we transmit a pulsed signal with a peak transmit power of P_t watts and a pulse length of τ seconds. Close examination of terms in the radar equation shows that the signal power per pulse depends on the product of P_t and τ, the latter through the scattering area A_{scatt}.

But instead suppose we can only generate $P_t/2$ watts. Note that if we double the pulse length τ to 2τ, the received signal power remains constant:

$$P_t \cdot \tau = \frac{P_t}{2} \cdot 2\tau$$

Perhaps this is obvious because the total energy per pulse remains constant. But this allows us to trade off transmitter peak power for pulse length, important for many practical reasons.

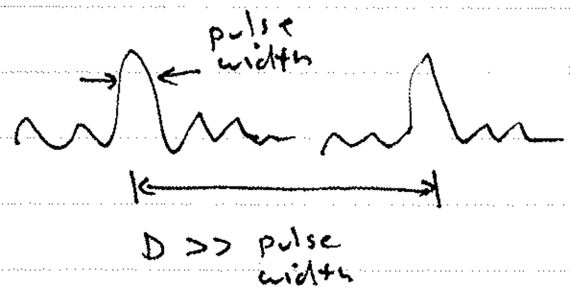
Can you identify some?

Resolution

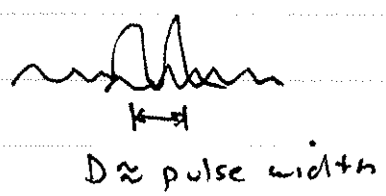
A very important parameter of any radar, but in particular an imaging radar, is its ability to discriminate closely-spaced targets. We can qualitatively define the resolution of a system as the minimum distance at which two objects are distinguishable. A strict quantitative description is quite difficult as many factors affect our ability to separate objects.

In an imaging system, we usually specify resolution in the range dimension (δ_r) and azimuth dimension (δ_{az}) separately, as they are dependent on entirely different sets of parameters.

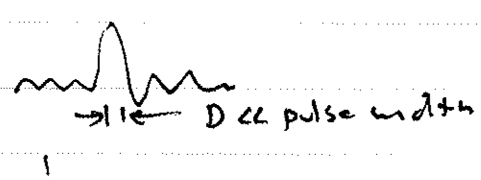
Suppose the impulse response of our radar in the range direction is a sinc function. Here are several situations:



Targets well separated



Targets may be distinguished



Targets merge into one

Following the argument suggested by the above, we can specify the resolution of a radar as about the value of the pulse width.

For a rectangular impulse response the resolution is simply the pulse length.

For a sinc impulse response, we'll choose the peak-to-null length, or half the null-to-null length. Some people multiply this by 0.89 to use the 3-dB width.

Since pulse lengths are defined in time, and we are usually interested in ground resolutions, we must convert from time to distance in the usual way.

$$\boxed{\delta_r = \frac{c\tau}{2\sin\theta}}$$ another one to memorize

Sometimes we want to consider the resolution in <u>slant</u> range, so we neglect the $\sin\theta$ dilution:

$$\delta_{slant} = \frac{c\tau}{2}$$

<u>Relation to bandwidth</u>

Consider a short pulse of length τ. Its spectrum $S(f)$ has an equivalent bandwidth of about

$$bw \approx \frac{1}{\tau}$$

so that we can relation resolution directly to signal bandwidth:

$$\delta_r = \frac{c}{2 \cdot bw \cdot \sin\theta}$$

$$\delta_{slant} = \frac{c}{2 \cdot bw}$$

Note that increasing signal bandwidth decreases resolution. This now provides a direct coupling between the system performance calculation of signal power and the system noise power (through $k T_{sys} B$).

Some system examples

Now that we have used many of the parameters of system design, it is instructive to look at a few real systems to begin to develop a feel for what range of system parameters we can design a practical system.

AIRBORNE SYSTEMS

Radar parameter	*JPL L-BAND*	*JPL C-BAND*	*Military high performance*
Frequency	1.275 GHz	5.3 GHz	10 GHz
Range bandwidth	20/40 MHz	20/40 MHz	up to 500 MHz
Peak transmit power	6000 W	1500 W	1000000 W
Pulse repetition rate	500 nominal	500 nominal	2000
Antenna dimensions	1.2 by 0.5 m	1.2 by 0.12 m	0.5 by 0.5 m
Antenna elevation beam width	6.2°	6°	3°
Operational range	15 km	15 km	200 km
Aircraft altitude	8 km	8 km	15 km
Look angles	20–65°	20–65°	20-89°
Ground range swath	12 km	12 km	1-15 km

SATELLITE SYSTEMS

Radar parameter	*SEASAT*	*ERS–1*	*JERS–1*
Frequency	1.275 GHz	5.3 GHz	1.275 GHz
Range bandwidth	19 MHz	15.55 MHz	15 MHz
Peak transmit power	1000 W	4800 W	1100–1500 W
Pulse repetition rate	1463–1647	1679 nominal	1505–1606
Antenna dimensions	10.7 by 2.16 m	11 by 1 m	12 by 2.2 m
Antenna elevation beam width	6.2°	6°	6.2°
Altitude decay, appr.	10 m/day	10 m/day	10 m/day
Satellite altitude	800 km	790 km	568 km
Look angles	20–26°	21–26°	35°
Ground range swath	100 km	100 km	85 km