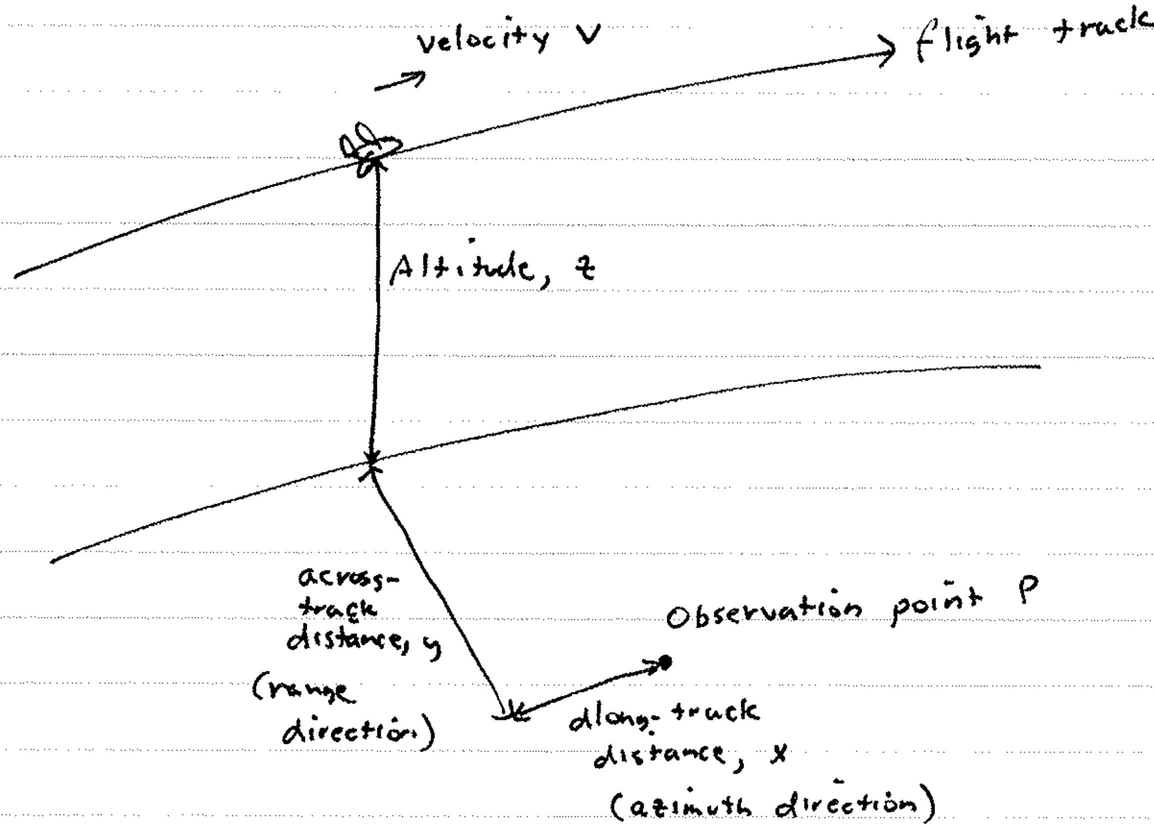
EE355/GP265

Imaging radar geometry

In order to calculate the many parameters needed to describe and process imaging radar data, it is important to have a well defined set of coordinates and definitions. We'll start with a few terms.



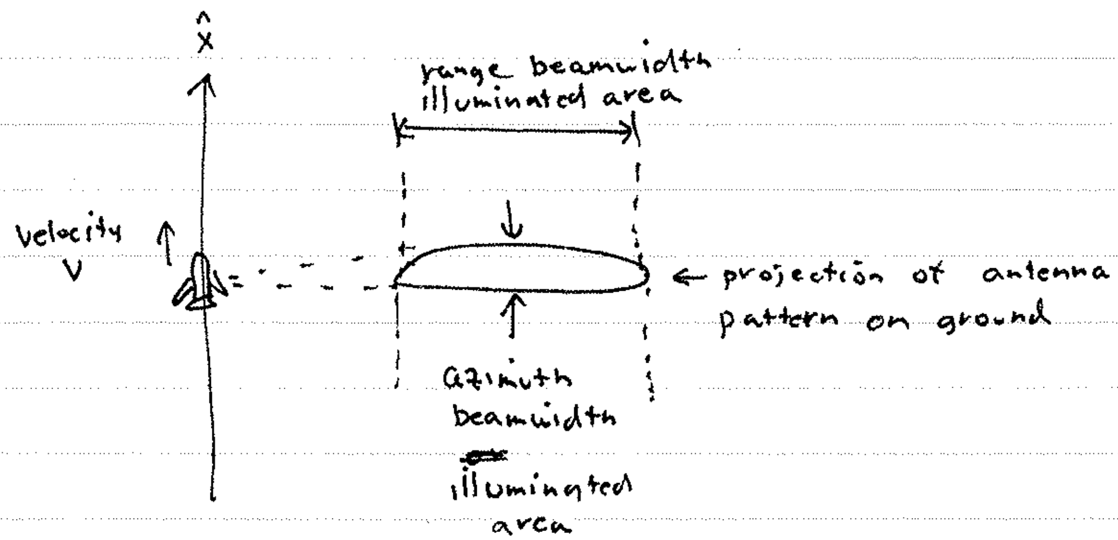
Generally we assume that the velocity v is in the x-direction. The range to a point on the ground at $(x, y, -z)$ relative to the radar is

$$r = \sqrt{x^2+y^2+z^2}$$

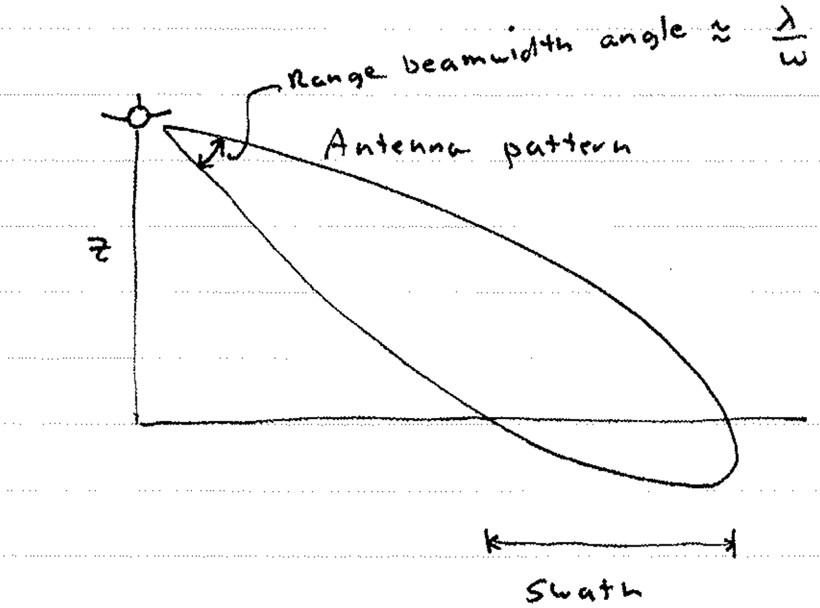
For truly side-looking geometries, $x=0$ and

$$r = \sqrt{y^2+z^2}$$

Let's see a top view

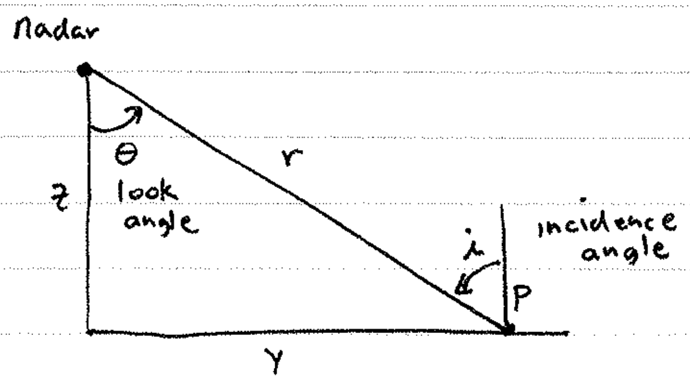


Rear view



The beamwidth of the range pattern is $\frac{\lambda}{w}$, where w is the width of the antenna. Similarly the azimuth beamwidth is $\frac{\lambda}{l}$, l being the length of the antenna.

Another rear view



For observing a point P on the surface, we define the angle from a vector ~~to~~ the point to a vector straight down (the nadir vector) as the look angle, denoted θ.

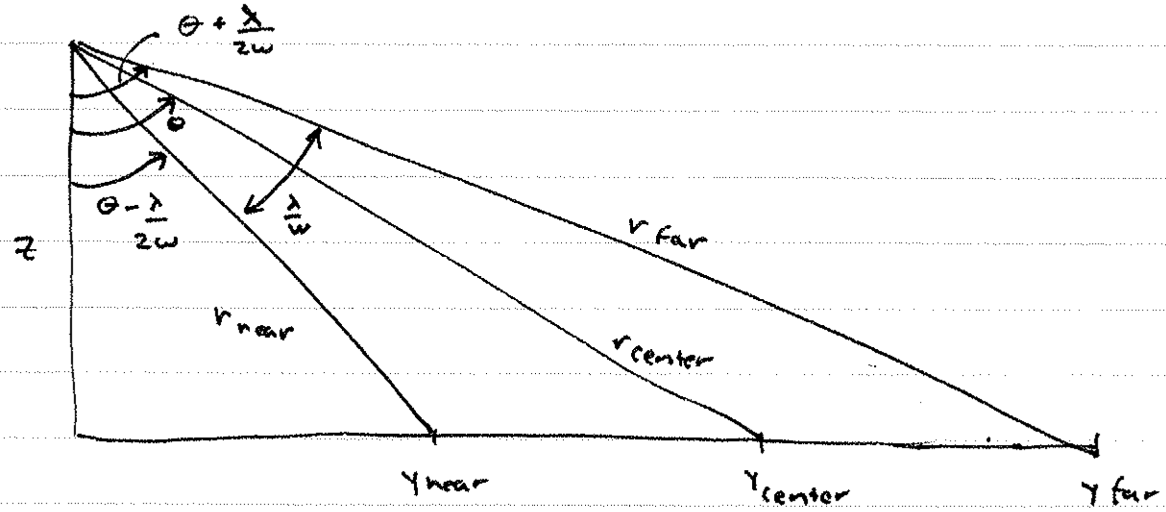
For an approximation neglecting the curvature of the Earth, the "flat-Earth approximation", the look angle is numerically equal to the incidence angle i. This holds fairly well for airborne platforms, but is not valid for spaceborne geometries where the curved Earth must be considered.

Useful relations:

$$\sin\theta = \frac{y}{r}$$
$$\cos\theta = \frac{z}{r}$$

Mapping a swath

Let's consider mapping a swath defined by the look angle to the center of the swath and by the range antenna beamwidth. The geometric construction we use is:



The distance y to the center of the swath is

$$y_{center} = \sqrt{r^2_{center} - z^2}$$

$$= r_{center} \sin\theta$$

$$= \frac{z \sin\theta}{\cos\theta} = z \tan\theta$$

Near and far swath distances:

$$y_{near} = z \tan\left(\theta - \frac{\lambda}{2w}\right)$$

$$y_{far} = z \tan\left(\theta + \frac{\lambda}{2w}\right)$$

So the swath width $y_{far} - y_{near} = z\left[\tan\left(\theta + \frac{\lambda}{2w}\right) - \tan\left(\theta - \frac{\lambda}{2w}\right)\right]$

A crude approximation for narrow swaths $\left(\frac{\lambda}{w} << 1\right)$ would be

swath width $= \dfrac{z \cdot \frac{\lambda}{w}}{\cos^2\theta}$ ← exercise left to student

We can now estimate the size of the illuminated area on the ground by computing the ~~product~~ product of the swath width and the azimuth beamwidth on the ground:

$$\text{Area} = \text{Swath width} \times \text{Azimuth size}$$

$$= \frac{z\frac{\lambda}{w}}{\cos^2\theta} \times \frac{r\lambda}{l}$$

Using values at the center of the swath for calculation

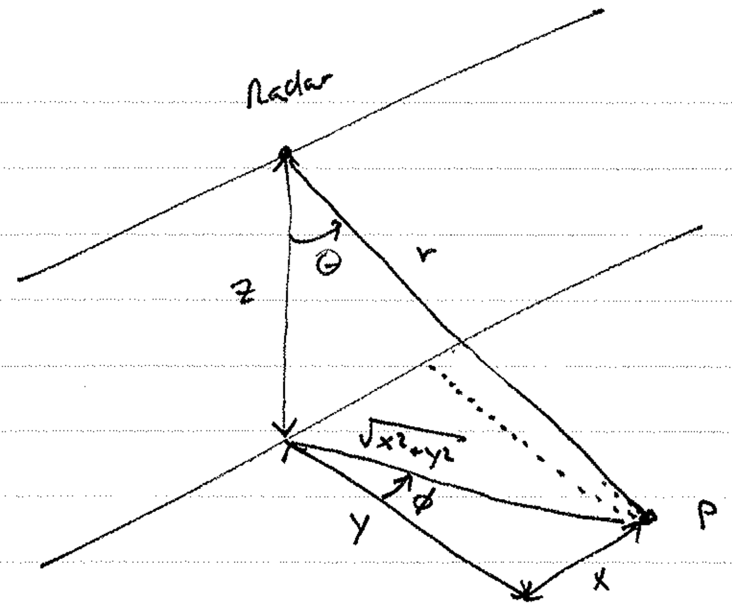
$$\text{Area} = \frac{r^2\lambda^2}{\cos\theta A_{ant}} = \frac{z^2\lambda^2}{\cos^3\theta\, A_{ant}}$$

We'll need this when we want to calculate SNR.

Other angles

The antenna for a radar system does not always point broadside to the platform. Sometimes this is intentional, as for forward-looking reconnaisance radars, and sometimes it is an unavoidable consequence of platform motions. The amount of deviation from broadside is usually called the squint angle, with positive values being forward squint and negative values backward squint.

The following geometric construction is useful:



The look angle, θ, is defined in the plane containing the radar and the observation point. The angle φ, in the surface plane, is the <u>squint angle</u>.

$$\sin\theta = \frac{\sqrt{x^2+y^2}}{r}$$

$$\cos\theta = \frac{z}{r}$$

$$\sin\phi = \frac{x}{\sqrt{x^2+y^2}}$$

Sometimes these two angles are combined in to a <u>composite squint angle</u>, which we'll denote <u>sq</u>,

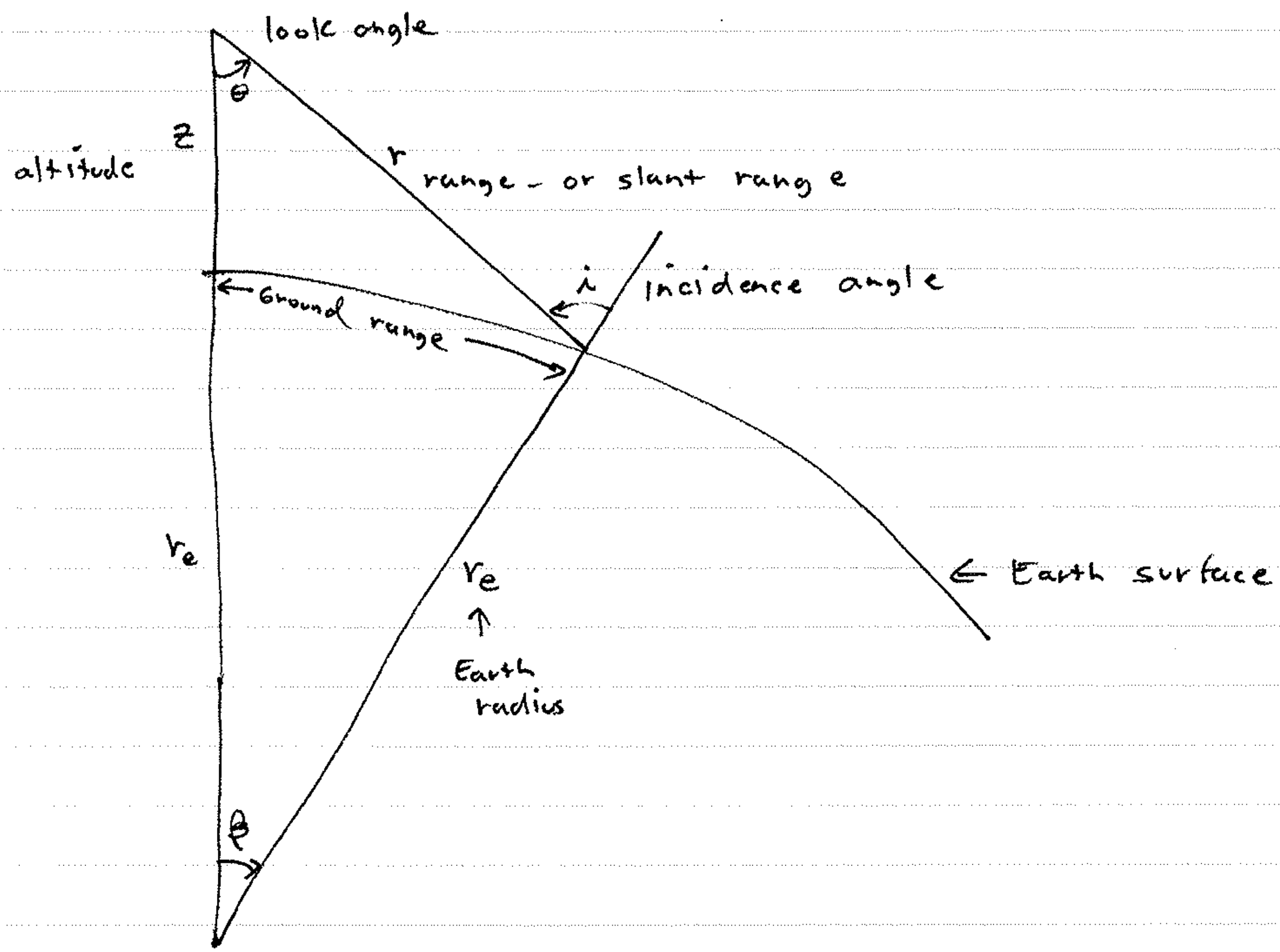
$$\sin(sq) = \frac{x}{r}$$

Note $\sin(sq) = \sin\theta \sin\phi$. We have to be careful which squint angle is referred to in any discussion.

Spherical Earth constructions

The above, while detailed, is rather obvious and easily derived. But for many cases of interest, such as for all spaceborne observations or even high-accuracy aircraft observations, we cannot neglect the fact that the Earth is round. This adds a complication to the geometry that is worth describing in more detail.

Imaging geometry from space:

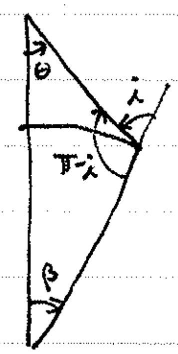


We can begin to relate these quantities from spherical trig relations. Using the laws of cosines and sines:

$$\cos\theta = \frac{r^2 + (z+r_e)^2 - r_e^2}{2\cdot r\cdot(z+r_e)}$$

$$\frac{\sin\beta}{r} = \frac{\sin\theta}{r_e}$$

These relations and similar ones suffice for most needed geometrical quantities. But let's examine the incidence angle more closely with the following distorted picture:



We know $\theta + \beta + (\pi - i) = \pi$

$$\boxed{\theta + \beta - i = 0}$$

Note that in the limit of $r_e \Rightarrow \infty$ (flat earth), $\beta \Rightarrow 0$ and we see $\theta - i = 0$, the above-used flat earth computation. Expressing the boxed equation as

$$\boxed{i = \theta + \beta}$$

we see that the incidence angle <u>exceeds</u> the look angle by the Earth-centered angle β. This distinction is important because

- The system parameters affect performance calculations and data acquisition are dependent on Θ

- The scattering phenomenology is dependent on i

so we must always keep in mind the difference between them.